Regular Ocean Waves (Linear Waves)

\[ \eta = \text{displacement of the water surface, varies with time and location, (meters, feet, etc.)} \]
\[ H = \text{wave height, constant with time at a given location, (m, ft)} \]
\[ a = \text{wave amplitude (m, ft), } a = H/2 \text{ for linear waves} \]
\[ L = \text{wave length (m, ft)} \]
\[ T = \text{wave period, time required for one complete wave to pass a fixed point (seconds, minutes, hours)} \]
\[ f = \text{wave frequency } = 1/T \text{ (cycles/sec } = \text{ Hertz, Hz)} \]
\[ \omega = \text{wave circular or radian frequency } = 2\pi/T = 2\pi f \text{ (radians/sec)} \]
\[ c = \text{wave celerity or wave speed, } c = L/T \text{ (m/sec, ft/sec, etc.)} \]
\[ c_g = \text{wave group velocity (m/sec, ft/sec, etc.)} \]
\[ d = \text{water depth (m, ft, fathoms (fm) = 6 ft)} \]
\[ k = \text{wave number } = 2\pi/L \text{ (m}^{-1}, \text{ft}^{-1}, \text{etc.)} \]
\[ u = \text{water particle velocity in the } x \text{ (horizontal) direction (m/sec, ft/sec, etc.)} \]
\[ w = \text{water particle velocity in the } z \text{ (vertical) direction (m/sec, ft/sec, etc.)} \]
\[ a_x = \text{water particle acceleration in the } x \text{ (horizontal) direction (m/sec}^2, \text{ft/sec}^2, \text{etc.)} \]
\[ a_z = \text{water particle acceleration in the } z \text{ (vertical) direction (m/sec}^2, \text{ft/sec}^2, \text{etc.)} \]

Wave Displacement \[ \eta = a \cos(kx - \omega t) \]
Wave Number \[ k = 2\pi/L \]
Radian Frequency \[ \omega = 2\pi/T \]
Dispersion Relation \[ \omega^2 = gk \tanh(kd) \]
<table>
<thead>
<tr>
<th></th>
<th>Shallow Water</th>
<th>Intermediate</th>
<th>Deep Water</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d/L &lt; 1/20, 0.05$</td>
<td>$0.05 &lt; d/L &lt; 1/2$</td>
<td>$d/L &gt; 1/2$</td>
</tr>
<tr>
<td></td>
<td>$d/(gT^2) &lt; 0.0025$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wave Speed</td>
<td>$c = \frac{L}{T} = \sqrt{gd}$</td>
<td>$c = \frac{L}{T} = \frac{\omega}{k} = \sqrt{\frac{g}{k} \tanh(kd)}$</td>
<td>$c = c_o = \frac{L}{T} = \sqrt{\frac{gL}{2\pi}}$</td>
</tr>
<tr>
<td>Wave Length</td>
<td>$L = T\sqrt{gd}$</td>
<td>$L = \frac{gT^2}{2\pi} \tan\left(\frac{2\pi d}{L}\right)$</td>
<td>$L = \frac{gT^2}{2\pi}$</td>
</tr>
<tr>
<td>Wave Period</td>
<td>$T = \frac{L}{\sqrt{gd}}$</td>
<td></td>
<td>$T = \sqrt{\frac{2\pi L}{g}}$</td>
</tr>
<tr>
<td>Group Velocity</td>
<td>$c_g = c$</td>
<td>$c_g = \frac{c}{2} \left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)}\right]$</td>
<td>$c_g = \frac{c}{2}$</td>
</tr>
</tbody>
</table>

### Horizontal Water Particle Velocity

- \[ u = \frac{agk \cosh(kz + kd)}{\omega} \cosh(kd) \cos(kx - \omega t) \]
- \[ u = a \omega \cosh(kz + kd) \cosh(kd) \cos(kx - \omega t) \]

### Vertical Water Particle Velocity

- \[ w = \frac{agk \sinh(kz + kd)}{\omega} \cosh(kd) \sin(kx - \omega t) \]
- \[ w = a \omega \sinh(kz + kd) \cosh(kd) \sin(kx - \omega t) \]

### Horizontal Water Particle Acceleration

- \[ a_x = agk \frac{\cosh(kz + kd)}{\cosh(kd)} \sin(kx - \omega t) \]

### Vertical Water Particle Acceleration

- \[ a_z = -agk \frac{\sinh(kz + kd)}{\cosh(kd)} \cos(kx - \omega t) \]

### Energy per unit surface area

- \[ E = \frac{1}{2} \rho g a^2 = \frac{1}{8} \rho g H^2 \]

### Energy per unit wave crest width

- \[ E_r = \frac{1}{2} \rho g a^2 L = \frac{1}{8} \rho g H^2 L \]

### Energy flux per unit wave crest width

- \[ \dot{E} = E_{c_g} = \frac{1}{2} \rho g a^2 c_g = \frac{1}{8} \rho g H^2 c_g \]