Network equilibrium models with battery electric vehicles

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Abstract

The limited driving ranges, the scarcity of recharging stations and potentially long battery recharging or swapping time inevitably affect route choices of drivers of battery electric vehicles (BEVs). When traveling between their origins and destinations, this paper assumes that BEV drivers select routes and decide battery recharging plans to minimize their trip times or costs while making sure to complete their trips without running out of charge. With different considerations of flow dependency of energy consumption of BEVs and recharging time, three mathematical models are formulated to describe the resulting network equilibrium flow distributions on regional or metropolitan road networks. Solution algorithms are proposed to solve these models efficiently. Numerical examples are presented to demonstrate the models and solution algorithms.

1. Introduction

Battery electric vehicles (BEVs) have enjoyed fast-growing adoption in recent years, thanks to the concern about climate change, the advancement of battery technologies and expeditiously rising prices of crude oil (e.g., Larminie and Lowry, 2003; Tamor et al., 2013; Feng and Figliozzi, 2013; He et al., 2013b; Gardner et al., 2013). However, those early adopters of BEVs do endure the inconvenience and cost incurred by the limited driving range of BEVs, insufficient charging infrastructure and long battery charging or swapping time (e.g., He et al., 2013a; Nie and Ghamami, 2013). The fear of batteries running out of power en route, normally referred to as range anxiety in the literature (see, e.g., Pearre et al., 2011), will inevitably affect BEV drivers’ travel choices. Although many governments are planning to deploy public charging stations in their regions, and a number of other strategies tackling range anxiety have emerged (e.g., He et al., 2013c), it seems unrealistic to expect that range anxiety can be eliminated in the near future.

Range anxiety is not just limited to BEV drivers. Indeed, drivers of other types of alternative-fuel vehicles often experience it. The literature of alternative-fuel vehicles has considered the potential need of recharging those vehicles to reach their destinations. For example, Kuby and Lim (2005) recognized their limited driving ranges and investigated locating refueling stations to ensure alternative-fuel vehicles to refuel more than once to successfully complete their entire trips. For the same purpose, Wang and Lin (2009) formulated the refueling logic of vehicles to be a system of linear equations. Both studies assume that travelers between an origin–destination (O–D) pair choose the shortest path, which is given and fixed. For electric vehicles, activity-based approaches have been proposed to investigate individual vehicle routing and scheduling problem with recharging in the literature (e.g., Kang and Recker, 2009, 2012; Schneider et al., 2012). Adler et al. (2013) defined a BEV shortest walk problem that finds the route between an O–D pair with minimum detouring, and proved the problem to be polynomially solvable. Note that the obtained shortest walk may include cycles for detouring to recharge...
batteries. In contrast to these previous studies, this paper investigates how the limited driving range and recharging requirement of BEVs affect their drivers’ route choices and subsequently the equilibrium flow distribution on regional or large metropolitan road networks where charging stations are few and far between. Among others, the most relevant studies in the literature include Jiang et al. (2012), and Jiang and Xie (2013). In the former, a network equilibrium model is formulated upon paths whose lengths are within the driving ranges of BEVs. The so-called path-constrained traffic assignment model can be solved efficiently by a solution algorithm proposed in the latter. Jiang et al. (2013) further extended the model to consider mixed gasoline and electric vehicular flows, and their combined choices of destination, route and parking subject to the driving range limit. All three studies do not consider recharging behaviors of BEV drivers and assume the energy consumption of BEVs is independent of traffic congestion.

In this paper, assuming that the energy consumption of a BEV is not affected by traffic congestion, i.e., the consumption is flow-independent, we first formulate a network equilibrium model that abides by the driving ranges of BEVs and accommodates their recharging decisions. An iterative solution procedure is proposed to solve the model efficiently. We further extend the model to consider the time required for recharging, which can be substantial, depending on the amount of recharged energy and the type of charging stations. Lastly, considering the potential impact of traffic congestion on the fuel economy of BEVs, we investigate a novel network equilibrium model with flow-dependent energy consumption.

For the remainder, Section 2 formulates a base model that describes network equilibrium with flow-independent energy consumption of BEVs, and then proposes a solution procedure. Section 3 extends the base model to consider the recharging time while Section 4 investigates another network equilibrium model with flow-dependent energy consumption. Section 5 presents numerical examples to demonstrate the proposed models and solution algorithms. Lastly, Section 6 concludes the paper.

2. Base model

2.1. Notation

We consider a regional or metropolitan road network. Let \( G(N, A) \) denote the network where \( N \) and \( A \) are the sets of nodes and links in the network respectively. We denote a link as \( a \in A \) or its starting and ending nodes i.e., \( a = (i, j) \in A \). Travel demands are between a set of O–D pairs, i.e., \( W \). Let \( g^w \) and \( P^w \) be the travel demand and the set of paths between O–D pair \( w \in W \) respectively. In addition, \( f^w_a \) represents the traffic flow on path \( p \in P^w \) of O–D pair \( w \in W \). We further denote \( a(w) \) as the origin node of O–D pair \( w \in W \), and \( \delta_{ap} \) is the path-link incidence, which equals 1 if path \( p \) traverses link \( a \in A \) and 0 otherwise. Let \( v_a \) and \( d_a \) be the traffic flow and distance of link \( a \). The travel time of link \( a \) is a strictly increasing function of the flow on the link, i.e., \( t_a(t_a) \). For example, the following Bureau of Public Roads (BPR) function can be used:

\[
t_a = t_a^0 \left[ 1 + 0.15 \left( \frac{v_a}{c_a} \right)^4 \right]
\]

where \( t_a^0 \) is the free-flow travel time of link \( a \), and \( c_a \) represents the capacity of link \( a \).

2.2. Definition and formulation of network equilibrium

It is assumed in this paper that all vehicles in the network are BEVs. This assumption is not necessarily restrictive as the models proposed below can be easily extended to accommodate both electric and regular vehicles. It is further assumed that a limited number of charging stations are located at certain nodes of the network, and thus vehicles traveling along a path may not pass by a charging station. We thus have the following definition:

**Definition 1.** A path is usable if a BEV is able to complete the path without or with recharging.

The distance of a usable path must be within the driving range of a BEV, if none charging station exists along the path. Otherwise, it is usable as long as the vehicle can recharge its battery at charging stations along the path to avoid running out of charge before reaching its destination. Fig. 1 is a simple example to further illustrate the definition.

The O–D pair 1–2 is connected by three paths, i.e., 1–2, 1–3–2 and 1–4–2. A charging station is located at both nodes 3 and 4. It is assumed that the battery size of the BEV is 24 kWh; its initial state of charge is 4 kWh and the energy consumption rate is 0.3 kWh/mi. It is easy to see that path 1–2 is not a usable path. Along the other two, the BEV can reach node 3 or 4.
without running out of charge. Because the vehicle can recharge at node 3 or 4 to achieve a driving range up to 80 miles if fully charged, it can reach the destination successfully. Therefore, both path 1–3–2 and path 1–4–2 are usable.

When traveling between their origins and destinations, it is reasonable to assume that BEV drivers select routes to minimize their travel costs, which may include electricity cost and travel time cost. Note that the former is much smaller than the latter. For example, considering an electricity price of $0.12 per kWh, a value of travel time of $20 per hour and a travel speed of 50 miles per hour, the electricity cost for a Nissan Leaf 2013 is 8.7% of its travel time cost (US Department of Energy, 2013). Therefore, we hereinafter simply adopt time minimization as the decision criterion for route choices. More specifically, we assume that travelers choose the paths with the least travel time among all the usable paths. Without loss of generality, we further assume that for each O–D pair, there exists at least one usable path. In our base model, we consider homogenous vehicles with the same battery size and initial state of charge, another assumption that can be easily relaxed. Moreover, it is assumed in Sections 2 and 3 that traffic congestion does not affect the fuel economy of BEVs. The energy consumption of a BEV thus depends on travel distance rather than travel time. The above assumptions and considerations yield the following network equilibrium:

**Definition 2.** At equilibrium, all the utilized paths are usable and the travel times of all the utilized paths of one O–D pair are the same, which are less than or equal to that of any unutilized usable paths of the same O–D pair.

We are now ready to construct our base model to describe the above network equilibrium. Because the energy consumption of a BEV is flow-independent, the usability of a path is also independent of traffic flow and can be pre-determined. Denote the set of all usable paths between O–D pair \( w \) as \( \hat{P}^w \) and we have the following formulation for network equilibrium (NE) with BEVs:

\[
\text{NE : } \begin{align*}
\min_f & \sum_{a \in A} \int_0^{s_a(A)} \sum_{w \in W} \sum_{p \in \hat{P}^w} f_p^w t_a(z) dz \\
\text{s.t.} & \sum_{p \in \hat{P}^w} f_p^w = g^w \quad \forall w \in W \\
f_p^w \geq 0 & \quad \forall p \in \hat{P}^w, w \in W
\end{align*}
\]  

(1)

where \( f \) is the path flow vector.

In the above, the constraints ensure flow balance between each O–D pair and the nonnegativity of path flows respectively. As compared to the classical formulation of Beckmann et al. (1956), NE requires the path flow of each unusable path to be zero. It is straightforward to verify the equivalency of NE to Definition 2 and that the equilibrium link flow is unique with the strictly-increasing travel time function previously assumed. Note that the path-constrained traffic assignment problem in Jiang et al. (2012) can be also formulated as NE, if \( \hat{P}^w \) includes the paths no longer than the driving range of BEVs.

### 2.3. Solution procedure

As formulated, NE is a convex program with linear constraints. If all the usable paths are enumerated beforehand, it can be solved easily by commercial nonlinear solvers such as CONOPT (Drud, 1994). Considering path enumeration is time-consuming, this section presents an iterative solution procedure. The procedure starts with a subset of \( \hat{P}^w, w \in W \), and solves a restricted version of NE defined upon the subset. Another sub-problem is then solved to determine whether the solution to the restricted NE solves the original formulation. If not, a new usable path will be generated and added to the subset and the iteration proceeds until termination.

A few new variables are introduced for formulating the sub-problem. For each node \( i \in N \), we use \( b_i \) to represent the upper limit of electricity that the node can provide for recharging. The variable equals 0 if there is no charging station at node \( i \); otherwise, it is a sufficiently large constant because the number of chargers at a charging station is assumed to be sufficient. Let \( L_{\text{max}} \) and \( L_0 \) be the battery size and initial state of charge. For a BEV traveling between O–D pair \( w \in W \), the recharging amount of electricity at node \( i \) is \( F_i^w \) and the state of charge at node \( i \) after recharging is \( L_i^w \). Let \( \Delta \) be the node-link incidence matrix associated with the network and \( E^w \) is a vector with a length of \( |N| \). The vector consists of two non-zero components: one has a value of 1 in the component corresponding to the origin of \( w \) and the other has a value of \(-1\) in the component corresponding to the destination of \( w \). Given the current link flow solution \((\cdots, \bar{v}_a, \cdots)\) to the restricted NE, the sub-problem, shortest usable path finding or SP, can be formulated as follows for each O–D pair \( w \in W \):

\[
\text{SP : } \begin{align*}
\min_{x,a} & \sum_{a \in A} t_a(\bar{v}_a)x^w_a \\
\text{s.t.} & \Delta x^w + E^w \\
L_j^w - L_i^w + d_{a\sigma} - F_j^w = & \rho_a^w \quad \forall (i,j) = a \in A \\
L_j^w - d_{a\sigma} \geq -M(1 - x^w_a) & \quad \forall (i,j) = a \in A
\end{align*}
\]  

(3)

(4)

(5)
\[-K(1-\lambda_w^a) \leq \rho_w^a \leq K(1-\lambda_w^a) \quad \forall (i,j) \in A\]  
\[0 \leq F_w^i \leq b_i \quad \forall i \in N\]  
\[0 \leq L_w^i \leq L_{\text{max}} \quad \forall i \in N\]  
\[I_w^i\]_{i \in W} = L_0\]  
\[\lambda_w^a \in \{0, 1\} \quad \forall (i,j) \in A\]  

where $K$ and $M$ are sufficiently large constants; $\sigma$ is the energy consumption rate of BEVs; $\lambda_w^a$ is a binary variable, which equals 1 if link $a$ is utilized and 0 otherwise; $\rho_w^a$ is a variable that equals 0 if link $a$ is utilized and is unrestricted otherwise.

In the above, the objective function is to minimize the total travel time. Constraint (2) ensures flow balance. Constraints (3) and (5) specify the relation between the states of charge of BEV batteries at the starting and ending nodes of any utilized link. Constraint (4) ensures that BEVs do not run out of charge on any utilized link. Constraint (6) suggests that BEVs can only recharge at nodes with a charging station while constraint (7) sets the upper and lower bounds of the states of charge of BEV batteries. Constraint (8) specifies the initial state of charge. Finally, constraint (9) requires $x_w^a$ to be binary.

SP is a mixed integer linear program, and can be easily solved by commercial solvers such as CPLEX 12.2 for small or medium sized problems. In addition, with a simple twist, the algorithm proposed by Jiang and Xie (2013) can be utilized to solve SP. More specifically, during the labeling process, if a node with a recharging station is selected to calculate the new labels, zero will be used as its distance label instead of its original distance label. Similar algorithms can also be found in Laporte and Pascoal (2011) and Adler et al. (2013) among others.

For each O–D pair $w \in W$, the optimal solution to SP, denoted as $(\ldots, x_w^a, \ldots, F_w^i, \ldots)$, can be used to construct a shortest usable path, i.e., $\bar{p}^w$. The iterative procedure of solving NE can thus be written as follows:

**Step 0:** For each O–D pair $w \in W$, solve SP with $(\ldots, \bar{b}_a, \ldots) = (\ldots, 0, \ldots)$. Construct $\bar{P}^w = \{\bar{p}^w\}$.

**Step 1:** Solve the restricted NE upon $\bar{P}^w$. Denote $(\ldots, \bar{v}_a, \ldots)$ and $(\ldots, \bar{\mu}^a, \ldots)$ as the optimal solutions and multipliers associated with constraint (1).

**Step 2:** For each O–D pair $w \in W$, solve SP. For $w \in W$, if $\mu^w > \sum_{a \in A} \bar{v}_a x_w^a$, add $\bar{p}^w$ into $P^w$. If $\mu^w \leq \sum_{a \in A} \bar{v}_a x_w^a$ for all O–D pairs, stop and $(\ldots, \bar{v}_a, \ldots)$ is the equilibrium link flow distribution; Otherwise, go to Step 1.

As the number of usable paths in a network is finite, the above procedure terminates in a finite number of steps.

### 3. Equilibrium model considering recharging time and range anxiety

The base model does not take into account the recharging time, which can be substantial, depending on the recharging amount and the power of the charger. For example, for a BEV with a 24 kWh battery, it may take 20 h to replenish a depleted battery at 1.2 kW power level. At 60 kW power level, 24 min are still needed (ETEC, 2010). See Table 1 for the charging parameters of BEV chargers that are currently deployed (Morrow et al., 2008; Dong et al., 2014).

The recharging time may affect motorists’ route and charging decisions. To see this, let’s revisit the example in Section 2.2. In Fig. 1, both paths 1–3–2 and 1–4–2 are usable and travelers thus choose between them. Assuming that their travel times are 25 and 20 min respectively due to different speed limits, travelers would prefer path 1–4–2, if the recharging time is not considered. If travelers consider the recharging time and aim to reach their destinations as quickly as possible, they do not necessarily fully recharge their vehicles at nodes 3 or 4, where the remaining level of charge of the battery is 1 kWh. To complete their trips, travelers only need to recharge 0.5 kWh at node 3 or 2 kWh at node 4. Suppose that it takes 10 min for the charging stations at nodes 3 and 4 to recharge 1 kWh of electricity, and the recharging time will be 5 and 20 min respectively. Travelers would thus prefer path 1–3–2 to 1–4–2, as the total time to complete the former is 30 min while 40 min for the latter. The route choice is thus different with or without considering charging time.

The base model does not consider range anxiety of drivers either. Due to the uncertainty of fuel economy, drivers of BEVs may not feel comfortable to fully deplete their batteries. Instead, they likely reserve a safety margin to hedge against variations of energy consumption, and would not allow the remaining battery range to fall below it. Franke et al. (2012) defined a comfortable range as the lowest remaining battery state of charge that a driver would experience comfortable or free from range anxiety, and then conducted a field study to evaluate it. Motivated by this previous study as well as the above example of recharging time, this section extends the base model to consider the impact of the comfortable range and recharging times on choices of route and recharging amount. It is assumed that BEV drivers attempt to minimize their trip times, which include travel times (more explicitly, driving times) and recharging times. Besides route choices, they also

<table>
<thead>
<tr>
<th>Charging level</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power (kW)</td>
<td>1.44</td>
<td>6</td>
<td>90</td>
</tr>
<tr>
<td>Charging circuit</td>
<td>120 V, 15 A</td>
<td>240 V, 30 A</td>
<td>50 V, 200 A</td>
</tr>
</tbody>
</table>
decide where and how much to recharge their vehicles to ensure the remaining battery state of charge no less than the comfortable range. Consequently a usable path previously defined becomes the one along which a BEV is able to complete without allowing the remaining battery state of charge to fall below the comfortable range. At network equilibrium, all the utilized paths of each O–D pair will be usable and yield the same trip times, which are less than or equal to that of any unutilized usable path of the same O–D pair.

To describe the equilibrium, let \( c_i(F_{iw}^w) \) denote the time it takes for a BEV to recharge \( F_{iw}^w \) amount of electricity at node \( i \in N \). For simplicity, we assume a linear charging time function, i.e., \( c_i(F_{iw}^w) = c_i^1 + c_i^2 F_{iw}^w \), where the first component \( c_i^1 \) represents the fixed time for the recharging activity; the second component is the variable time and \( c_i^2 \) depends on the type of chargers as shown in Table 1. Without loss of generality, we assume that there is at most one charging station at each node with the same type of chargers. If there is no charging station at node \( i \), both \( c_i^1 \) and \( c_i^2 \) are zero. Note that by adding another cost component to the charging function, one can consider the impact of different prices for recharging at different types of charging stations. In this paper, we assume the same unit price of electricity at all charging stations.

Let \( s_{iw}^w \) represent the minimal time that a BEV of O–D pair \( w \in W \) needs to spend on recharging activities when traversing a path \( p \in P^w \). Because of the flow independence of the energy consumption of BEVs, it is straightforward to examine each usable path to determine optimal charging locations and amounts to compute \( s_{iw}^w \). As the charging time function is linear, \( s_{iw}^w \) can be uniquely specified. The network equilibrium with recharging time (NE-RT) can thus be formulated as follows:

\[
\begin{align*}
\text{NE-RT} : \\
&\min w \sum_{i \in A} w \sum_{p \in \mathcal{P}_i} F_{iw}^w t_a(z) dz + \sum_{w \in W} w \sum_{p \in \mathcal{P}_p} s_{iw}^w F_{iw}^w \\
&\text{s.t. } (1)-(2)
\end{align*}
\]

The above formulation is another convex program with linear constraints and commercial nonlinear solvers such as CONOPT can solve it globally. Similarly, it is straightforward to introduce multiple classes of drivers to consider different comfortable ranges, representing different levels of range anxiety. Indeed, one numerical example in Section 5 considers three classes of drivers.

In the above, the objective function is to minimize the total trip time that includes the driving time, i.e., \( \sum_{a \in \mathcal{A}} w t_a(x_a^w) + \sum_{i \in N} w (c_i^1 r_i^w + c_i^2 F_i^w) \). Constraint (5a) dictates that the remaining battery range is always no less than the comfortable range en-route. Constraints (11) and (12) ensure that \( r_i^w \) equals 1 if BEVs recharge at node \( i \) and 0 otherwise, as the objective function is to minimize \( c_i r_i^w \). Similarly, SP-RT is another mixed integer linear program, and can be easily solved by commercial solvers such as CPLEX 12.2 for small or medium instances. However, because it encapsulates both route and recharging decisions, the labeling process to solve SP discussed in Section 2.3 is not readily applicable to solve SP-RT.

Let \((\cdots, \hat{x}_a^w, \cdots, \tilde{r}_i^w, \cdots, \tilde{F}_i^w, \cdots)\) denote the optimal solution to SP-RT for O–D pair \( w \in W \). Based on \( \hat{x}_a^w \), we can easily construct the usable path with minimal trip time, i.e., \( \hat{p}^w \). In addition, \( \tilde{r}_i^w \) and \( \tilde{F}_i^w \) dictate the optimal recharging plan along the path, and the minimal recharging time is \( s_{iw}^w = \sum_{i \in N} (c_i^1 \hat{r}_i^w + c_i^2 \tilde{F}_i^w) \). For completeness, we present the following solution procedure to NE-RT:

**Step 0:** For each O–D pair \( w \in W \), solve SP-RT with \((\cdots, \hat{v}_a, \cdots) = (\cdots, 0, \cdots)\). Construct \( \hat{p}^w = \{\hat{p}^w\} \) and compute \( s_{iw}^w \).

**Step 1:** Solve the restricted NE-RT upon \( \hat{p}^w \). Denote \((\cdots, \hat{v}_a, \cdots)\) and \((\cdots, \mu^w, \cdots)\) as the optimal solutions and multipliers associated with constraint (1).
Step 2: For each O–D pair \( w \in W \), solve SP-RT. For \( w \in W \), if \( \mu^w > \sum_{a \in A} t_a (u_a) x_{aw}^w + \sum_{l \in L} (c_1^l f^w_l + c_2^l f^w_l) \), add \( \tilde{p}^w \) into \( \tilde{P}^w \). If \( \mu^w \leq \sum_{a \in A} t_a (u_a) x_{aw}^w + \sum_{l \in L} (c_1^l f^w_l + c_2^l f^w_l) \) for all O–D pairs, stop and \((\cdots, \tilde{v}_a, \cdots)\) is the equilibrium link flow distribution; Otherwise, go to Step 1.

4. Network equilibrium with flow-dependent energy consumption

In the above models, the energy consumption of BEVs depends only on the distance traveled. Bigazzi et al. (2012) investigated the effect of traffic congestion on the fuel economy of BEVs and found that BEVs may become more fuel efficient as the average speed increases, particularly at local arterials. Although it is difficult to foresee how future developments in the battery and vehicular technologies may enhance the fuel economy of BEVs at various traffic conditions, we assume in the section that the energy consumption for a BEV to traverse a link increases as the travel time increases. This implies that the fuel economy of BEVs becomes flow-dependent. So is the usability of a path between an O–D pair.

4.1. Definition of network equilibrium

With the flow-dependent usability of paths, the network equilibrium described in Definition 2, does not necessarily exist. To see this, consider a single O–D network connected by two parallel links. The O–D travel demand is 5.5, and the travel time functions for two links are 1 + \( v_1 \) and 0.5 + 2\( v_2 \) while their lengths are 1 and 0.5 respectively. Suppose that the initial state of both links is 1.6 and the energy consumption at each link is the function of distance and travel time, i.e., \( e_a = 0.8 - d_a + 0.2 t_a \). Although there is no charging station in the network, both paths are usable when they are in free-flow conditions. Consequently, three possible scenarios emerge at equilibrium, i.e., only link 1 or 2 is utilized or both links are utilized. It is easy to verify that if only either link 1 or 2 is utilized, the increased link travel time will cause the link unusable. If both links are utilized, according to Definition 2, they will have the same travel times. We thus obtain \( v_1 = 3.5, v_2 = 2, t_1 = t_2 = 4.5, e_1 = 1.7 \) and \( e_2 = 1.3 \). Apparently, link 1 becomes unusable because \( e_1 = 1.7 > 1.6 \). This contradicts to Definition 2, suggesting that the defined equilibrium does not exist in this network. This implies that with flow-dependent energy consumption, the utilized usable paths do not necessarily have the same travel times when the network achieves equilibrium.

We then claim that \( v_1 = 3 \) and \( v_2 = 2.5 \) is an equilibrium flow pattern, under which travel times and energy consumptions are \( t_1 = 4, t_2 = 5.5, e_1 = 1.6 \) and \( e_2 = 1.5 \). One can verify that under such a flow pattern, no drivers have incentives to switch paths unilaterally. Specifically, drivers of path 1 have no interest to change to path 2 for a longer travel time. Reversely, drivers of path 2 have no interest to switch to path 1 either, because doing so would make path 1 unusable and thus compromise the chance of completing their trips.

The above example motivates us to investigate a more general definition of network equilibrium with flow-dependent path usability. Several new variables are introduced for this purpose. For each path \( p \), we divide it into \( n_p \) sub-paths, denoted as \( p_{pq}, q = 1, \ldots, n_p \), based on the locations of charging stations along the path. Each sub-path starts and ends at a charging station, origin or destination, and there is no additional charging station along a sub-path. The link energy consumption \( e_a(d_a, t_a) \) is assumed to be strictly increasing with the distance \( d_a \) and the travel time \( t_a \). For model simplicity, we do not consider recharging time and comfortable range in this section. It is further assumed that each BEV starts its trip with a fully charged battery. Note that these two assumptions are not restrictive and can be relaxed. Below we introduce a concept of charging-depleting paths to facilitate the presentation of our idea:

Definition 3. A charge-depleting path is a usable path with the energy consumption of a BEV reaching exactly its full battery capacity between two adjacent charging stations along the path.

Recall that \( L_{\text{max}} \) represents the battery size. The above definition suggests that along a charge-depleting path, there exists at least a sub-path where the energy consumption, i.e., \( e_{pq} = \sum_{d_a \in D} e_a(d_a, t_a) \), is equal to \( L_{\text{max}} \).

Definition 4. A flow pattern is in user equilibrium if all the utilized paths are usable; the travel times of utilized regular or non-charge-depleting paths are equal and no larger than that of any unutilized usable path; the travel times of utilized charge-depleting paths may be less than or equal to that of a utilized non-charge-depleting path.

Mathematically, the user equilibrium conditions can be written as follows:

\[
\begin{align*}
  & \text{If } f^w_p > 0, \text{ then } e_{pq} \leq L_{\text{max}} \quad \forall p, q, w \quad (13) \\
  & \text{If } f^w_p > 0, \text{ and } \exists q \text{ where } e_{pq} = L_{\text{max}}, \text{ then } t_p \leq \mu^w \quad \forall p, w \quad (14) \\
  & \text{If } f^w_p > 0, \text{ and } \forall q, e_{pq} < L_{\text{max}}, \text{ then } t_p = \mu^w \quad \forall p, w \quad (15) \\
  & \text{If } f^w_p = 0, \text{ and } \forall q, e_{pq} \leq L_{\text{max}}, \text{ then } t_p \geq \mu^w \quad \forall p, w \quad (16)
\end{align*}
\]

where \( t_p \) is the travel time of path \( p \).

Condition (13) ensures that all the utilized paths are usable. (14) and (15) specify that the travel time of each utilized path of O–D pair \( w \in W \) equals \( \mu^w \), except for the charge-depleting paths whose travel times can be less than or equal to \( \mu^w \). (16) dictates that the travel time of any unutilized usable paths is greater than or equal to \( \mu^w \). Note that if we denote \( \mu^w \) as the
equilibrium travel cost, for any utilized charge-depleting path \( p \), according to (14), there must exist a variable, say, \( \beta_p \geq 0 \) such that \( \gamma_p + \beta_p = \mu_p \). In a way, the variable \( \beta_p \) may be interpreted as the cost of range anxiety that arises when the energy consumption reaches the limit. When the energy consumption is flow-dependent, route choices of travelers affect not only the path travel time but also the path usability. When choosing paths, travelers would consider both effects, and the variable \( \beta_p \) may capture their sense of risk for being running out of charge along the path. However, we caution that such an interpretation lacks a behavioral basis and it may not be appropriate to compare “the costs of range anxiety” associated with different charge-depleting paths.

4.2. Model formulation and solution algorithm

The problem of finding a user equilibrium flow pattern as defined in Definition 4 can be formulated as the following nonlinear complementarity problem (NCP), which we call network equilibrium with flow-dependent energy consumption or NE-FD:

The NCP is equivalent to finding \( (\cdots, f_{p}^{w}, \cdots, \gamma_{pq}^{w}, \cdots) \in \Pi \) that solves the following variational inequality (VI):

\[
\sum_{p,w} \left[ \sum_{a} \delta_{a,p} \bar{t}_{a} + \sum_{q=1}^{n_{p}} \gamma_{pq}^{w} \left( f_{p}^{w} - L_{\max} + \hat{e}_{p} \right) \right] \left( \begin{array}{c} f_{p}^{w} - f_{p}^{w} \\ L_{\max} - \hat{e}_{p} \end{array} \right) \geq 0, \quad \forall \left( \cdots, f_{p}^{w}, \cdots, \gamma_{pq}^{w}, \cdots \right) \in \Pi
\]
To avoid path enumeration, an iterative solution procedure similar to that in Section 2.3 can be adopted to generate paths as the procedure proceeds. The path generation sub-problem is similar to SP. The only difference is to add to SP additional cuts that correspond to charge-depleting paths under the current flow solution because travel times of those paths are likely to be less than the equilibrium cost of the O–D pair. More specifically, if we denote $\tilde{p}$ as one of the charge-depleting paths under the current flow solution, one valid cut can be written as $\sum_{a} \delta_{a,p} x_{a} \leq \sum_{a} \delta_{a,\tilde{p}} - 1$.

5. Numerical examples

In this section, we present numerical examples to demonstrate the proposed models. We first solve NE-RT for the Sioux Falls network (see Fig. 2), which consists of 24 nodes, 76 links and 552 O–D pairs. Table 2 reports the free-flow travel time and capacity of each link. The O–D demands are downloaded from Bar-Gera (2013). The link distances are assumed to be 2.5 times of the link free-flow travel times. The battery capacity, i.e., $L_{\text{max}}$, is set as 24 kWh, consistent with the battery size of a Nissan Leaf (Nissan USA, 2013). The energy consumption rate $\omega$ is 0.29 kWh/mi, as per the EPA fuel economy of Nissan Leaf 2013 (US Department of Energy, 2013). The comfortable range, i.e., $m^c$, is set as zero. Other parameters include $c_1 = 5$ min, $L_0 = 0.2 L_{\text{max}}$, and $c_i^l = \begin{cases} 41.67, & \text{Level 1} \\ 10, & \text{Level 2} \\ 0.67, & \text{Level 3} \end{cases}$ min/kWh. There exist five charging stations in the network, i.e., two level-1 stations at nodes 11 and 15, two level-2 at nodes 5 and 16, and one level-3 at Node 12. We adopt CONOPT and CPLEX 12.2 to solve NE-RT and SP-RT respectively, and the equilibrium link flows are reported in Table 3.
In order to analyze the impact of the initial state of charge of battery on the equilibrium and charging station utilization, we also solve NE-RT with another two initial states of $L_0 = 0.25L_{\text{max}}$ and $0.35L_{\text{max}}$, in addition to $0.2L_{\text{max}}$. Fig. 3 compares the average utilizations, i.e., average numbers of recharging trips, of three types of charging stations in these three cases. It can be observed that as the initial state of charge increases, the utilizations of all three types of charging stations decrease. In addition, level 3 station is always the most popular, followed by level 2 and then level 1, which makes intuitive sense as higher-power stations offer less amount of recharging time and are thus more favored by travelers.

### Table 2
Link capacity ($10^3$ veh/h) and free-flow travel time (min).

<table>
<thead>
<tr>
<th>Link</th>
<th>Free-flow travel time</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–2</td>
<td>3.6</td>
<td>6.02</td>
</tr>
<tr>
<td>1–3</td>
<td>2.4</td>
<td>9.01</td>
</tr>
<tr>
<td>2–1</td>
<td>3.6</td>
<td>12.02</td>
</tr>
<tr>
<td>2–6</td>
<td>3</td>
<td>15.92</td>
</tr>
<tr>
<td>3–1</td>
<td>2.4</td>
<td>46.83</td>
</tr>
<tr>
<td>3–4</td>
<td>2.4</td>
<td>34.22</td>
</tr>
<tr>
<td>3–12</td>
<td>2.4</td>
<td>46.81</td>
</tr>
<tr>
<td>4–3</td>
<td>2.4</td>
<td>25.82</td>
</tr>
<tr>
<td>4–5</td>
<td>1.2</td>
<td>28.25</td>
</tr>
<tr>
<td>4–11</td>
<td>3.6</td>
<td>9.04</td>
</tr>
<tr>
<td>5–4</td>
<td>1.2</td>
<td>46.85</td>
</tr>
<tr>
<td>5–6</td>
<td>2.4</td>
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<td>5–9</td>
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<td>6–2</td>
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<td>9.92</td>
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<td>9.90</td>
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<tr>
<td>6–8</td>
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<td>21.62</td>
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<tr>
<td>7–8</td>
<td>1.8</td>
<td>15.68</td>
</tr>
<tr>
<td>7–18</td>
<td>1.2</td>
<td>46.81</td>
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<tr>
<td>8–6</td>
<td>1.2</td>
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<td>8–7</td>
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<td>15.68</td>
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<td>8–9</td>
<td>2</td>
<td>10.10</td>
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<td>9–5</td>
<td>3</td>
<td>10.90</td>
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<tr>
<td>9–6</td>
<td>1.2</td>
<td>20</td>
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<tr>
<td>9–8</td>
<td>2</td>
<td>10.10</td>
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<td>9–10</td>
<td>1.8</td>
<td>27.83</td>
</tr>
<tr>
<td>10–9</td>
<td>1.8</td>
<td>27.83</td>
</tr>
</tbody>
</table>

In order to analyze the impact of the initial state of charge of battery on the equilibrium and charging station utilization, we also solve NE-RT with another two initial states of $L_0 = 0.25L_{\text{max}}$ and $0.35L_{\text{max}}$. In addition to $0.2L_{\text{max}}$. Fig. 3 compares the average utilizations, i.e., average numbers of recharging trips, of three types of charging stations in these three cases. It can be observed that as the initial state of charge increases, the utilizations of all three types of charging stations decrease. In addition, level 3 station is always the most popular, followed by level 2 and then level 1, which makes intuitive sense as higher-power stations offer less amount of recharging time and are thus more favored by travelers.

### Table 3
Equilibrium link flow (veh/h).

<table>
<thead>
<tr>
<th>Link</th>
<th>Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–2</td>
<td>2100</td>
</tr>
<tr>
<td>1–3</td>
<td>8500</td>
</tr>
<tr>
<td>2–1</td>
<td>2500</td>
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<tr>
<td>2–5</td>
<td>4100</td>
</tr>
<tr>
<td>3–1</td>
<td>8100</td>
</tr>
<tr>
<td>4–3</td>
<td>19000</td>
</tr>
<tr>
<td>5–4</td>
<td>8100</td>
</tr>
<tr>
<td>6–2</td>
<td>23000</td>
</tr>
<tr>
<td>6–5</td>
<td>23000</td>
</tr>
<tr>
<td>7–8</td>
<td>23000</td>
</tr>
<tr>
<td>8–1</td>
<td>23000</td>
</tr>
<tr>
<td>9–5</td>
<td>23000</td>
</tr>
<tr>
<td>10–9</td>
<td>23000</td>
</tr>
</tbody>
</table>

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The average recharging frequency, amount and time in all three cases are depicted in Fig. 4. It can be observed that the highest average recharging frequency is 0.32 when $L_0 = 0.2 L_{\text{max}}$, suggesting that on average a BEV recharges 0.32 times to complete its trip. Correspondingly, the recharging amount is 0.97 kWh, which takes 7 min on average. As the initial state of charge increases, the average recharging frequency, amount and time all decrease. Particularly, when the initial state equals $0.35 L_{\text{max}}$, both the recharging amount and time are almost zero. All these observations suggest that the current driving ranges of BEVs are sufficient in the example network, if their batteries are reasonably charged.

To further demonstrate NE-RT, we solve it for the Sioux Falls network with three different classes whose comfortable ranges are two, one and zero kWh respectively. For each O–D pair, all three classes have the same travel demand and initial battery range, i.e., $0.3 L_{\text{max}}$. The solution procedure terminates within approximately 30 min, using a personal computer with Intel (R) Core (TM) Duo 2.8 GHz CPU and 6 GB RAM. This demonstrates the potential of NE-RT to be applied to networks of realistic size. Fig. 5 compares the recharging information of three travel classes. As expected, travelers with higher range-anxiety tolerance choose to recharge less often, and thus their average recharging amount and time are less, as compared to those with a lower tolerance.

![Fig. 3. Charging station utilisations.](image-url)

![Fig. 4. Recharging information.](image-url)
Considering flow-dependent energy consumption, we solve NE-FD for the Nguyen–Dupius network shown in Fig. 6. This network consists of 13 nodes, 19 links, and four O–D pairs. The link characteristics and O–D demands can be found in Nguyen and Dupius (1984). We assume that the link distance is 1.5 times of the link free-flow travel time. The initial battery state and battery capacity are both equal to 16.56 kWh. The link energy consumption function is \( e_a(d_a, t_a) = 0.174d_a + 0.116t_a \). Two charging stations are located at nodes 6 and 11, respectively. During the solution iterations, we adopt PATH (Ferris and Munson, 1999) to solve NE-FD upon a subset of paths, which are generated by solving SP with cuts using CPLEX 12.2. With different initial solutions, two equilibrium link flow patterns are obtained and reported in Table 4. It can be observed from Table 4 that these two flow patterns are substantially different. For instance, the flow on link 17 changes from 1.99 to 13.81. The non-uniqueness and possible substantial variation of equilibria will inevitably create challenges for predicting BEV traffic flow distribution in the case of flow-dependent energy consumption. Depending on specific policy goals, different
strategies may be applied. For example, from the perspective of robust policy-making, one may examine and design against the worst-case equilibrium flow pattern (e.g., Lou et al., 2010). Tables 5 and 6 present the detailed path information of these two flow patterns, including path travel time, flow and sub-path energy consumption. It can be observed from Table 5 that under the first flow pattern, there are six utilized paths in total, of which only path 6 is charge-depleting. However, its travel time is the same as path 5 and thus the cost of range anxiety is zero. In contrast, under the second flow pattern in Table 6, there are two different path travel times for O–D pair 4–2, and three for O–D pair 4–3. Indeed, paths 5, 7 and 8 are all charge-depleting and incur costs of range anxiety, as reported in the table.

6. Conclusions

We have investigated the network equilibrium problems with battery electric vehicles. Considering the limited driving range and the recharging need of electric vehicles, we assume that drivers of electric vehicles select paths to minimize their
driving times while ensuring not running out of charge. We then define the network equilibrium conditions and formulate them into a mathematical program. An iterative procedure is proposed to solve the program to find the equilibrium flow pattern. We further extend the model to consider the recharging time and flow-dependent energy respectively. More specifically, the first extension successfully models travelers’ recharging decisions and captures the impact of recharging time and range anxiety on travelers’ route choices. With the energy consumption being flow-dependent, the second extension captures travelers’ considerations on how their route choices impact both the travel time and usability of paths. We analyze in the numerical examples the utilization for different types of charging station, and the average recharging frequency, amount and time under various initial states of charge. It is observed that regardless of initial state of charge, travelers always favor higher-power charging stations, and the current BEV battery size may be sufficient for trips across the example network. In addition, we consider heterogeneous drivers with various levels of range anxiety and compare their recharging behaviors. It is observed that range anxiety increases travelers’ recharging frequency, amount and time. Finally, the equilibrium flow pattern is shown to be non-unique if the energy consumption is flow-dependent.

Note that the modeling framework in this paper is trip-based and overlooks potential connections among different trips conducted by the same traveler. One way to this limitation is to incorporate activity-based analyses (e.g., Recker, 1995, 2001; Lam and Yin, 2001; Gan and Recker, 2008; Kang and Recker, 2013), which, however, will impose computational challenges. Our future study will attempt to find a right balance between the computational tractability and model realism. Note that we only solve the network equilibrium problem with flow-dependent energy consumption on a small network. The computational difficulty lies in solving the NCP or VI formulation efficiently at each iteration. Our future study will investigate the possibility of applying new techniques, such as linear approximation (see, e.g., Lawphongpanich et al., 2014), to further enhance computational efficiency. Built upon the network equilibrium models proposed in this paper, our future study will also investigate how to optimally locate charging stations at traffic networks.

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