Integrated pricing of roads and electricity enabled by wireless power transfer

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A B S T R A C T

This paper explores integrated pricing of electricity and roads enabled by wireless power transfer technology. We envision that high-power, high-efficiency wireless power transfer technologies are mature in the near future, which electrify roads to be charging infrastructures. The prices of electricity at electrified roads will affect electric vehicles’ route choices while the energy requirement of those vehicles will in return affect the operations of the power network and thus the prices of electricity. To determine the optimal prices of electricity and roads to maximize social welfare, first- and second-best pricing models are proposed under different authoritarian regimes. More specifically, assuming that a government agency manages both transportation and power systems, we develop the first-best pricing model, based on which a marginal-cost pricing scheme is derived. The second-best pricing model is proposed if the agency participates in a competitive wholesale power market while being able to impose tolls on electrified roads. The toll design is formulated as a mathematical program with complementarity constraints, and is solved by a manifold suboptimization algorithm. Numerical examples are presented to offer insights on integrated pricing of roads and electricity and demonstrate its effectiveness on improving social welfare.

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1. Introduction

Electric vehicles have been long recognized as a promising way to reduce traffic emissions locally and petroleum dependence. Early models of electric vehicles all came with limitations and costs that prevented them from competing with gas-fueled cars. However, recent advances in battery technologies and expeditiously rising prices of crude oil have helped relaunch electric vehicles. Many governments also provide a variety of subsidies or incentives to promote the adoption of electric vehicles. With the deployment of charging or battery swapping stations and further improvement of battery technologies, a fast-growing adoption of electric vehicles can be expected (e.g., Larminie and Lowry, 2003; Pearre et al., 2011; Dong and Lin, 2012; Sharma et al., 2012; Tamor et al., 2013; Feng and Figliozzi, 2013).

Deployment of electric vehicles will lead to more frequent and profound interaction between transportation and power systems. The policies implemented in the transportation system will change spatial and temporal distributions of electric vehicles and thus the pattern of their energy requirement, thereby affecting the operations of the power system. On the other hand, the provision of charging infrastructures and the associated charging expenses will affect travel patterns of electric vehicles and consequently the operations of the transportation system. The level of interaction of these two systems will largely depend on the market penetration of electric vehicles, advancement in charging technologies and, more importantly,
innovative control strategies that leverage those advanced charging technologies to foster the integration of these two systems. For example, under currently-available uncontrolled charging, i.e., let people charge their cars at will, the interaction of these two systems may be loose. However, under controlled charging and discharging via smart grid technologies, the interaction of these two systems can be much more profound.

Remarkable progress has been made in the field of wireless power transfer and recent research advances have enabled transferring power across large air gaps with high efficiency (e.g., Imura and Hori, 2011; Duong and Lee, 2011). It is proposed that electric vehicles in motion can be charged wirelessly (e.g., Imura et al., 2009; Madawala and Thrimawithana, 2011). The Energy Dynamics Laboratory at Utah State University recently proved that enough power can be transferred wirelessly to safely and effectively charge electric vehicles. Future versions of their system are expected to wirelessly charge vehicles at a speed of 75 mph (EDL, 2011). The wireless charging system is embedded in the pavement and thus transforms roads into public charging facilities. Such a pervasive, wireless charging platform will provide great opportunities to further integrate the transportation and power networks.

This paper represents one of the first attempts to analyze transportation and power networks coupled by electric vehicles. We envision in this paper that high-power, high-efficiency wireless power-transfer technologies are mature in the near future, which would be safe and feasible charging infrastructures. The electrified roads essentially become nodes of the power network, and the prices of electricity at electrified roads will directly influence electric vehicles’ route choices because those vehicles can be recharged while traversing the roads. On the other hand, the energy requirement of those vehicles will affect the operations of the power network and potentially the prices of electricity. In this paper, we develop mathematical models to determine optimal prices of electricity and roads to maximize social welfare associated with the coupled networks, under different authoritarian regimes.

For the remainder, Section 2 describes modeling considerations for both transportation and power networks. Assuming that both networks are in the jurisdiction of the same government agency, Section 3 discusses a first-best pricing model. Section 4 considers a second-best situation where the agency is only responsible for managing the road network while participating in a competitive wholesale power market. The toll design for electrified roads is formulated as a mathematical program with complementarity constraints, solved by a proposed solution algorithm based on manifold suboptimization. Section 5 presents a numerical example to demonstrate the pricing models and then conducts sensitive analyses on the market penetration of electric vehicles and the level of road electrification. The potential of the manifold suboptimization algorithm for solving more realistic problems is also demonstrated with another numerical example. Lastly, Section 6 concludes the paper.

2. Basic considerations and notations

In this pilot analysis, we make some rather restrictive assumptions to facilitate the model formulation to derive insights (some of the assumptions will be relaxed later in the paper for sensitive analyses). The assumptions include: (i) all vehicles in a road network are electric vehicles; (ii) all links in the road network are capable of wireless charging, and each link is connected to a bus in the power grid; (iii) when an electric vehicle is traveling on an electrified road, its energy consumption is fully provided by that particular road. In other words, vehicles do not use any power from their batteries; (iv) the energy consumption at a road is equal to a constant consumption rate multiplied by the travel time of the road. The corresponding energy cost or charging expense is equal to the price of electricity at the link multiplied by the energy provided to the vehicle, and lastly (v) when selecting routes from their origins to destinations, drivers minimize their total travel costs that may consist of travel time, road congestion toll and energy cost (charging expense).

Let GT(N,A) denote a regional (freeway) network, where N and A are the sets of nodes and links in the network respectively. We denote a link as a ∈ A or the pair of its starting and ending nodes, i.e., a = (i,j) ∈ A. Travel demands are originated from a set of origin nodes R ⊆ N, and travel to a set of destination nodes S ⊆ N. We denote the travel demand for each origin–destination (OD) pair as qrs, r ∈ R, s ∈ S, which is fixed and given.

Let A be the node-link incidence matrix associated with the network and EΣ is a vector with a length of |N|. The vector consists of two non-zero components: one has a value of 1 in the component corresponding to origin r and the other has a value of −1 in the component corresponding to destination s. We denote the flow between the OD pair (r,s) at link a as xia. Let va be the aggregate link flow at link a, and the travel time of the link ta is a strictly increasing function of va, e.g., in a form of the following BPR (Bureau of Public Roads) function:

\[ t_a = t^0_a + 0.15 \left( \frac{v_a}{c_a} \right)^4 \]

where \( t^0_a \) is the free-flow travel time for link a and \( c_a \) is the capacity of link a.

The set of feasible flow distributions of the transportation network can be written as follows:

\[ v_a = \sum_n x_{ia} \quad \forall a ∈ A \quad (1) \]

\[ Δx_{ia} = E^r q^r_a \quad \forall r ∈ R, \quad s ∈ S \quad (2) \]

\[ x_{ia} ≥ 0 \quad \forall a ∈ A, \ r ∈ R, \ s ∈ S \quad (3) \]
In the above, Eq. (1) defines the aggregate link flow; (2) ensures the flow balance between origin and destination nodes in the network and (3) is a non-negativity constraint.

We use $K$ to denote the set of buses in the transmission grid that serves the electrified roads and other regular loads. Let $GP(K,L)$ denote the power network, where $L$ is the set of transmission lines in the network. For the generator at bus $k \in K$, $g_k$ is the real power injection; $LG_k$ and $UG_k$ represent the lower and upper real power limit, respectively and $l_k$ is the regular real power load at bus $k$, which is given.

We further denote a transmission line or branch as the pair of its starting and ending buses, i.e., $(k,m) \in L$. For each branch, let $f_{km}$ represent the real power flowing in the branch and $U_{km}$ represent the thermal limit of real power flow. Let $B_{km}$ denote the inverse of the pu reactance and $\delta_k$ represents the multiplication of the base apparent power and voltage angle (in radians). Finally, we use $C$ to denote the set of the pairs of electrified roads and their serving buses.

The following constraints can be written to represent the operations of the power transmission network:

$$g_k - l_k - \sum_{(k,a) \in C} v_a t_a e_a - \sum_{km \in L} f_{km} = 0 \quad \forall k \in K$$ (4)

$$f_{km} = B_{km}(\delta_k - \delta_m) \quad \forall km \in L$$ (5)

$$- U_{km} \leq f_{km} \quad \forall km \in L$$ (6)

$$f_{km} \leq U_{km} \quad \forall km \in L$$ (7)

$$LG_k \leq g_k \quad \forall k \in K$$ (8)

$$g_k \leq UG_k \quad \forall k \in K$$ (9)

$$\delta_1 = 0$$ (10)

where $e_a$ is the energy consumption rate of an electric vehicle at link $a$. $v_a t_a e_a$ is thus the total energy requirement from electric vehicles traveling on link $a$. Constraints (4)–(10) are essentially from the standard DC optimal power flow model in a structural pu form (Sun and Tesfatsion, 2010). More specifically, (4) is the nodal power balance constraint; (5) is the linear expression of the real power branch flow equation, which is commonly seen in power systems textbooks, except that we multiply both sides by the base apparent power. Constraints (6)–(9) ensure the feasibility of the real power flow and power injection. Lastly, (10) sets the voltage angle at the reference node 1 to be 0.

3. First-best pricing

For benchmarking, we first consider a first-best condition where a government agency controls the operations of both the regional transportation and power transmission networks, and aims to maximize social welfare associated with both networks, e.g., minimizing the sum of the total travel time cost and generation cost of electricity. Note that other cost items, such as vehicular emissions can also be included without difficulty. The following system optimum model can be solved for the first-best condition in the coupled network:

SO:

$$\min_{(x,s,f,u)} \sum_k c_k(g_k) + \omega \sum_a t_a(v_a) v_a$$

s.t. (1) – (10)

where $\omega$ is the value of travel time and $c_k(g_k)$ is the total cost of generating $g_k$ amount of electricity at bus $k$, which is assumed to be an increasing and strictly convex function with $g_k$. In the above, the objective function is to minimize the total social cost, including the total generation cost of electricity and travel time cost of the transportation system respectively.

The KKT conditions of SO are as follows:

$$\left\{ \begin{array}{l}
\omega t_a + \left[ (\omega + \lambda_k e_a)(v_a \frac{dt_a}{dv_a}) + \lambda_k e_a t_a \right] - \rho_i^a + \rho_j^a = 0 \\
\forall a = (i,j), k, r, s, (k,a) \in C
\end{array} \right. \quad \forall k$$ (11)

$$\omega t_a + \left[ (\omega + \lambda_k e_a)(v_a \frac{dt_a}{dv_a}) + \lambda_k e_a t_a \right] - \rho_i^a + \rho_j^a \geq 0 \\
\forall a = (i,j), k, r, s, (k,a) \in C$$ (12)

$$\tau_k(g_k) - \lambda_k - \kappa_k^a + \kappa_k^b = 0 \quad \forall k$$ (13)

$$\kappa_k^a(g_k - LG_k) = 0 \quad \forall k$$ (14)

$$\kappa_k^b(g_k - UG_k) = 0 \quad \forall k$$ (15)

$$\kappa_k^a, \kappa_k^b \geq 0 \quad \forall k$$ (16)
The cost of travel on both routes between OD 1–3 is the same and equal to $51.75, the majority of which is travel time cost. As expected, the first-best pricing decentralizes the system optimum flow distribution to be tolled user equilibrium. The full functions and link flows, their marginal-cost tolls are slightly different, due to different serving buses in the power grid. As marginal-cost road toll is $2.325, $2.346 and $3.856 respectively. Note that although links 1–2 and 2–3 have the same cost electrified road, SO satisfies the Abadie constraint qualification and thus the KKT conditions hold (Bazaraa et al., 2006). For a constraint (6),

\[ U_{km} = 300 \text{MW}; B_{km} = 4 \text{ pu}; UG_2 = UG_4 = 600 \text{ MW}; UG_1 = UG_3 = 0; LG_6 = 0; e = 20.4 \text{ KWh/h} \text{ and } \omega = 10/\text{h}. \]

It can be observed that the optimal price of electricity is $85, $70, $75 and $80 at buses 1, 2, 3 and 4 respectively and the marginal-cost road toll is to charge \( \tau_{12} = $2.325 \), \( \tau_{23} = $3.856 \) at each link of the transportation network. The prices can be obtained by solving the above SO formulation.

\[ \begin{align*}
\lambda_k - \sigma_{km} - \theta_{km}^e + \theta_{km}^b &= 0 \quad \forall k, m \\
\theta_{km}^e(U_{km} + U_{km}) &= 0 \quad \forall k, m \\
\theta_{km}^b(U_{km} - U_{km}) &= 0 \quad \forall k, m \\
\theta_{km}^e - \theta_{km}^b &
\]
general network, we can linearize constraint (4) by assuming that the energy consumption at a road will be equal to a constant consumption rate multiplied by the distance of the road, instead of the travel time. Accordingly, the following constraint replaces (4) in SO:

\[ g_k - h_k - \sum_{(k,a) \in C} v_a d_a e_a - \sum_{km \in L} f_k m = 0 \quad \forall k \in K \quad (4a) \]

where \( d_a \) is the length of link \( a \). With all the constraints being linear, the Abadie constraint qualification is satisfied and thus the KKT conditions are both necessary and sufficient for SO. However, the resulting full travel cost becomes \( \omega t_a + \lambda_k e_a d_a + [\cdot \omega(t_d t_a/dv_a)] \), where the marginal external energy cost component, i.e., \( \lambda_k e_a d_a/dv_a \), vanishes, implying that the pricing signal does not capture the impact of road congestion on the power operation.

As formulated, SO is a regular nonlinear program. Commercial nonlinear solvers such as CONOPT (Drud, 1994) can solve it efficiently in our numerical experiments.

4. Second-best pricing

In this section, we consider a more practical second-best scenario where the government agency, e.g., traffic authority, only manages the transportation network. For the power network, we assume a competitive market consistent with the one proposed by the U.S. Federal Energy Regulatory Commission (FERC, 2003). In the market, an independent system operator (ISO) undertakes the daily operations of the transmission grid using locational marginal pricing. More specifically, ISO accepts supply and demand bids submitted by market participants, i.e., buyers and generators, and is responsible for determining the power commitments (supplies) to meet the demands, with an objective of minimizing power generation cost while ensuring the system security. In this case, the price of electricity ISO charges at each location will be the locational marginal price. Based on their locational marginal prices, the buyers pay ISO for the dispatched power (e.g., Sun and Tesfatian, 2010). Although the traffic authority has no control over the power market, the agency can participate in the wholesale market as a buyer. It will pay ISO, at locational marginal prices, for the dispatched power used to charge electric vehicles. On the other hand, the agency can adjust the retail price of electricity at each link to affect drivers’ route choice. In this case, the price of electricity essentially functions as a road toll (Palma and Lindsey, 2011). We are then interested in finding an optimal pricing strategy to maximize social welfare associated with the transportation system.

4.1. Model formulation

To compute the locational marginal price at the wholesale power market, we adopt the standard DC power flow model, which is a linear approximation to the AC models (e.g., Litvinov et al., 2004). Although the approximation leads to some loss of accuracy, the results match fairly closely with the full AC solutions (Overbye et al., 2004). The standard DC power flow model is written as follows:

DC:

\[
\begin{align*}
\min_{(g,f,\lambda)} & \sum_{a \in A} c_a(g_k) \\
\text{s.t.} & \quad (4a), (5)–(10)
\end{align*}
\]

The location marginal price is the Lagrangian multiplier associated with (4a) in the above model. Its KKT conditions consist of (4a)–(10) and (13)–(22), as shown in He et al. (2013).

As aforementioned, the traffic authority will determine the retail price at each road and use it as an instrument to manage traffic demand. Finding the optimal pricing scheme can be formulated as the following integrated price design or IPD problem:

IPD:

\[
\begin{align*}
\min_{(p,x,y,t,a,k,m,v,\omega,\lambda)} & \omega \cdot \sum_{a \in A} t_a(p_a) v_x + \sum_{(k,a) \in C} v_a \lambda_k e_a d_a \\
\text{s.t.} & \quad (1)–(3), (4a), (5)–(10) \text{ and (13)–(22)} \\
& \quad \left( \omega t_a + p_a d_a e_a - \rho_{i}^{a} + \rho_{r}^{a} \right) \cdot x_{ij} = 0 \quad \forall (i,j) \in A, r, s \quad (23) \\
& \quad \omega t_a + p_a d_a e_a - \rho_{i}^{a} + \rho_{r}^{a} \geq 0 \quad \forall (i,j) \in A, r, s \quad (24) \\
& \quad \sum_{a} v_a p_a d_a e_a \geq \sum_{(k,a) \in C} v_a \lambda_k e_a d_a \quad (25)
\end{align*}
\]

where \( p_a \) is the retail price of electricity at each link, determined by the traffic authority.

In the above, the objective function is to minimize the social cost for the transportation system, which consists of two components, i.e., total system travel time and the energy consumption, i.e., the electricity cost that the transportation authority pays to ISO at a price equal to the locational marginal price \( \lambda_k \). Constraints (1)–(3), (23) and (24) ensure that the traffic flow pattern is in user equilibrium, i.e., the utilized paths for the same OD pair will have the same generalized...
travel cost, which is less than or equal to those of unutilized paths of the same OD pair. Constraints (4a)–(10) and (13)–(22) are the KKT conditions of the standard DC power flow model, which honor the operational constraints of the power network while ensuring a least-cost power dispatch to meet the demands, including those from electric vehicles. Constraint (25) ensures nonnegative revenue for the government, i.e., without subsidy from the government, the total revenue from the retail of electricity should not be less than the payment to ISO.

4.2. Solution algorithm

As formulated, IPD is a mathematical program with complementarity constraints or MPCC, a class of problems difficult to solve for two reasons. The first is that the feasible region is non-convex and the second is some basic constraint qualifications such as Mangasarian–Fromowitz Constraint Qualification are violated (see, e.g., Chen and Florian, 1995; Scheel and Scholtes, 2000). Many (see, e.g., Luo et al., 1996 and references cited therein) have proposed special algorithms to solve them. Some only work well for small and medium problems (e.g., Li et al., 2012; Wang et al., 2013) while others, especially those based on solving equivalent nonlinear programs (e.g., Fletcher and Leyffer, 2004; Lawphongpanich and Yin, 2010) can handle larger problems. In this paper, we extend the manifold suboptimization algorithm proposed by Lawphongpanich and Yin (2010) to solve IPD. Conceptually, this algorithm relies on solving a sequence of nonlinear programs that are restrictions of the original MPCC. To tailor the manifold suboptimization algorithm to IPD, we define ten new sets, \( \Omega_{x_m}, \Omega_{p,m}, \Omega_{a,f}, \Omega_{a,n}, \Omega_{b,f}, \Omega_{b,n}, \Omega_{a,K}, \Omega_{b,K} \) and \( \Omega_{x,K} \) with \( \Omega_{x,x} \cup \Omega_{x,m} = A; \Omega_{a,f} \cup \Omega_{a,n} = \Omega_{b,f} \cup \Omega_{b,n} = L; \Omega_{a,K} \cup \Omega_{a,n} = \Omega_{b,K} \cup \Omega_{b,n} = K \). Based on those sets, a restricted version of IPD, i.e., RSPID, can be formulated as follows:

\[
\text{RSPID: } \min_{(p,x,v,g,f,\delta,\rho,\kappa,\sigma,\theta,x,x)} \omega \cdot \sum_{a} (\nu_a) v_a + \sum_{(k,m) \in C} \nu_a \delta_a e_a
\]

s.t. (1)–(3), (4a), (5), (10), (13), (17), (21)–(22), (25)

\[
\omega t_a + p_a d_a e_a - \rho^a \leq 0 \quad \forall a = (i,j) \in \Omega_{x,m}, r, s,
\]

\[
x^a = 0 \quad \forall a = (i,j) \in \Omega_{x,x}, r, s
\]

\[
\omega t_a + p_a d_a e_a - \rho^a \geq 0 \quad \forall a = (i,j) \in A \quad \text{and} \quad \not \in \Omega_{x,x}, r, s
\]

\[
x^a \geq 0 \quad \forall a = (i,j) \in A \quad \text{and} \quad \not \in \Omega_{x,x}, r, s
\]

\[
g_k - L_{K_k} = \quad \forall k \in \Omega_{K_k}
\]

\[
\kappa_k^a = \quad \forall k \in \Omega_{K_k}
\]

\[
g_k - L_{K_k} \geq 0 \quad \forall k \in \Omega_{K_k}
\]

\[
\kappa_k^b = \quad \forall k \in \Omega_{K_k}
\]

\[
f_{km} + U_{km} = 0 \quad \forall k \in \Omega_{a,f}
\]

\[
\theta^a_{km} = 0 \quad \forall k \in \Omega_{a,f}
\]

\[
f_{km} + U_{km} \geq 0 \quad \forall k \in \Omega_{a,f}
\]

\[
\theta^a_{km} \geq 0 \quad \forall k \in \Omega_{a,f}
\]

\[
f_{km} - U_{km} = 0 \quad \forall k \in \Omega_{b,f}
\]

\[
\theta^b_{km} = 0 \quad \forall k \in \Omega_{b,f}
\]

\[
f_{km} - U_{km} \geq 0 \quad \forall k \in \Omega_{b,f}
\]

\[
\theta^b_{km} \geq 0 \quad \forall k \in \Omega_{b,f}
\]

Note that by assigning the active set for each complementarity constraint, RSPID becomes a regular nonlinear program without complementarity constraints. What we explore next is to adjust the active set for each complementarity constraint to achieve a decreasing objective function while maintaining the feasibility of original IPD. The steps of the algorithm can be described as follows:

Step 0: Obtain a feasible solution \((p,x,v,g,f,\delta,\rho,\kappa,\sigma,\theta,x,x)\) to IPD. Set

\[
n = 1. \Omega^1_{x,m} = \{(i,j) = a = A : x^a = 0\}, \Omega^1_{p,m} = \{(i,j) = a = A : \omega t_a + p_a d_a e_a - \rho^a \leq 0\},
\]

\[
\Omega^1_{a,f} = \{(km) = L : f_{km} + U_{km} = 0\}, \Omega^1_{a,n} = \{(km) = L : \theta^a_{km} = 0\}, \Omega^1_{b,f} = \{(km) = L : f_{km} - U_{km} = 0\},
\]

\[
\Omega^1_{b,K} = \{(km) = L : \theta^b_{km} = 0\}, \Omega^1_{a,K} = \{(k) = K : g_k - L_{K_k} = 0\}, \Omega^1_{b,n} = \{(k) = K : \kappa_k^a = 0\}, \Omega^1_{b,K} = \{(k) = K : g_k - U_{K_k} = 0\}
\]
and $\Omega_{o,x}^1 = \{k \in K, \kappa_k^i = 0\}$.

**Step 1:** Set
\[
\Omega_{x,s} = \Omega_{o,x}^s, \Omega_{p,r,s} = \Omega_{o,p,r,s}^s, \Omega_{b} = \Omega_{o,b}^s, \Omega_{a} = \Omega_{o,a}^s, \Omega_{a,b} = \Omega_{o,a,b}^s, \Omega_{a,g} = \Omega_{o,a,g}^s, \Omega_{a,x} = \Omega_{o,a,x}^s, \Omega_{b,g} = \Omega_{o,b,g}^s
\]
and $\Omega_{b,x} = \Omega_{o,b,x}^s$. Let $(p^*(x), x^*, v^*, g^*, f^*, \theta^*, \rho^*, \kappa^*, \sigma^*, \theta^*, \varphi^*, \tau^*, \xi^*)^T$ solve RSIPD.

**Step 2:** Let
\[
\Delta_{A,x}^s = \left\{ (i,j) = a \in (\Omega_{x,s}^s \setminus \Omega_{p,r,s}^s) : \delta_{i,k,a}^s < 0 \right\}, \Delta_{A,p,r,s}^s = \left\{ (i,j) = a \in (\Omega_{x,s}^s \setminus \Omega_{p,r,s}^s) : \delta_{i,k,a}^s < 0 \right\},
\]
where $\delta_{i,k,a}^s, \delta_{i,k,b}^s, \delta_{i,k,c}^s, \delta_{i,k,d}^s, \delta_{i,k,e}^s, \delta_{i,k,f}^s, \delta_{i,k,g}^s, \delta_{i,k,h}^s, \delta_{i,k,i}^s$ and $\delta_{i,k,j}^s$ are the multipliers associated with constraints \((27), (26), (38), (39), (42), (43), (30), (31), (34)\) and \((35)\). If $\Delta_{A,x}^s = \Delta_{A,p,r,s}^s = \Delta_{A,f}^s = \Delta_{A,b}^s = \Delta_{A,g}^s = \Delta_{A,x}^s = \Delta_{A,b}^s = \Delta_{A,k}^s = \emptyset$, stop and $(p^*(x), x^*, v^*, g^*, f^*, \theta^*, \rho^*, \kappa^*, \sigma^*, \theta^*, \varphi^*, \tau^*, \xi^*)^T$ is strongly stationary. Otherwise, do the following and go to step 1:

(a) If $\Delta_{A,x}^s \cap \Delta_{A,p,r,s}^s = \emptyset$:
Set $\Omega_{x,s}^{n+1} = \Omega_{x,s}^n - \Delta_{A,x}^n$ and $\Omega_{p,r,s}^{n+1} = \Omega_{p,r,s}^n - \Delta_{A,p,r,s}^n$.

(b) If $\Delta_{A,f}^s \cap \Delta_{A,b}^s = \emptyset$:
Set $\Omega_{f}^{n+1} = \Omega_{f}^n - \Delta_{A,f}^n$ and $\Omega_{b}^{n+1} = \Omega_{b}^n - \Delta_{A,b}^n$.

(c) If $\Delta_{A,g}^s \cap \Delta_{A,k}^s = \emptyset$:
Set $\Omega_{g}^{n+1} = \Omega_{g}^n - \Delta_{A,g}^n$ and $\Omega_{k}^{n+1} = \Omega_{k}^n - \Delta_{A,k}^n$.

(d) If $\Delta_{A,a}^s \cap \Delta_{A,x}^s = \emptyset$:
Set $\Omega_{a}^{n+1} = \Omega_{a}^n - \Delta_{A,a}^n$ and $\Omega_{x}^{n+1} = \Omega_{x}^n - \Delta_{A,x}^n$.

(e) If $\Delta_{A,b}^s \cup \Delta_{A,k}^s = \emptyset$:
Set $\Omega_{b}^{n+1} = \Omega_{b}^n - \Delta_{A,b}^n$ and $\Omega_{k}^{n+1} = \Omega_{k}^n - \Delta_{A,k}^n$.

(f) Set $n = n + 1$.

Step 0 requires an initial and feasible solution to IPD, according to which the initial ten active sets are determined. To find such a feasible solution, we solve the following convex mathematical program:

\[
\min_{x,y} \int_0^T \omega \cdot t(x) dy + \sum_k c_k(g_k)
\]

s.t. \((1) - (3), (4a)\) and \((5) - (10)\);
The feasible solution to IPD can be easily constructed from the solution to the above program and with the retail price of the electricity being the locational marginal price at the link. At Step 1, RSIPD is solved based on the constructed active sets. Step 2 demonstrates the way to adjust the active sets. More specifically, for each class of complementarity constraints, we examine whether the KKT multipliers of the constraints corresponding to the intersection of those two active sets are negative or not. For example, at iteration n, we split constraint (23) into constraints (26) and (27) according to the pair of active set \( \Omega_{x,n}^p \cap \Omega_{p,x}^n \). Subsequently, we examine every element, i.e., \( a \), in the set of \( \Omega_{x,n}^p \cap \Omega_{p,x}^n \) to check whether the KKT multipliers \( d_{rs,x,a} \) and \( d_{rs,p,a} \), which are associated with constrains \( x_{rs,a} = 0 \) and \( v_{ta} + p_{ed} e_a - \rho_{i}^{rs} + \rho_{j}^{rs} = 0 \) respectively, are negative or not. If only one is negative, we remove it from the active set. If both, the one associated with a smaller multiplier is deleted. The procedure ensures the feasibility of the solution at each iteration and terminates in a finite number of iterations at a strongly stationary solution. The proof can be found in Lawphongpanich and Yin (2010).

5. Numerical examples

5.1. A small network example

The SO and IPD problems were solved for a coupled network that we created based on a nine-node road network and a subset of the IEEE 118-bus system (http://moctor.ece.iit.edu/Data) as shown in Fig. 2. The power network is composed of 16 transmission lines (undirected links) and 12 buses (nodes). There are 9 nodes, 18 links and 4 OD pairs in the transportation network. As shown by the solid square in Fig. 2, each link is connected to one particular bus in the power network, through which the energy consumption of electric vehicles on that link creates power load to the power grid. Table 1 lists the free-flow travel time, distance and capacity of each link. The OD matrix is shown in Table 2. Table 3 presents the thermal limit of real power flow in each branch and the associated B value, i.e., the inverse of the pu reactance. The total generation cost function at each bus was assumed as follows:

\[
c_k(g_k) = a_{k,2} \cdot g_k^2 + a_{k,1} \cdot g_k + a_{k,0}
\]

where \( a_{k,2} \) ($/MW^2h$), \( a_{k,1} \) ($/MWh$), and \( a_{k,0} \) ($/h$) are coefficients, which are given in Table 4 as well as the lower and upper limits on real power production of each generator. For the DC power flow model, the base apparent power was set as 100 MVA. The value of travel time \( \alpha \) and charging rate \( e \) equal to $10/h$ and 0.37 KWh/mi respectively.

We first consider the first-best situation. The SO model was directly solved by CONOPT to obtain the marginal-cost road tolls as reported in Table 5. The resulting optimal social cost is $363,897 and Table 6 presents the power injection and locational marginal price at each bus.

![Fig. 2. A coupled transportation and power network.](image-url)
Due to different extent of traffic congestion, marginal-cost tolls of the transportation network in Table 5 vary from being free to charging $14.43. Similarly, congestion in the transmission power network leads to spatial disparity of locational marginal price shown in Table 6.
For the second-best situation, we implemented the manifold suboptimization algorithm using GAMS (Brooke et al., 1992) to determine the integrated price design. CONOPT was used to directly solve RSIPD and the multipliers generated worked well. The optimal social cost in the transportation system is $375,846. For comparison, we also computed a scenario where the government agency does not intervene, and thus electric vehicles pay a price equal to locational marginal price. The resulting social cost at the transportation network under such a scenario is $410,379, implying that our pricing scheme reduces the social cost by $34,533, an 8.4% reduction.

The optimal retail prices of electricity are presented in Table 7 as well as the corresponding traffic flow distribution. Table 8 shows the power injection and locational marginal price at each bus. Caused by different route choices by electric vehicles, the prices in Table 8 are evidently different from those in Table 6.
It can be observed from Table 8 that the locational marginal price at the wholesale market is pretty stable, ranging from $16.60 to $21.64/MWh. In contrast, the optimized retail price of electricity in Table 7 presents a substantial spatial disparity, varying from zero to $477.29/MWh. Although it may not be appealing to the public, such a disparity is not rare in the power wholesale market. For example, Lewis (2010) examined locational marginal prices in Michigan, US, for two years. The maximal hourly locational marginal price varies within the state from $240.28 to $729.61 in the first year and $424.37 to $542.10 in the second year. To reduce the disparity, one can impose an additional constraint to IPD to bound the retail price of electricity, which, however, may lead to less social cost saving.

5.2. Sensitive analyses

In this section, we relax a couple of assumption made previously for the model formulation. More specifically, instead of assuming all the vehicles in a road network are electric vehicles and all links in the road network are capable of wireless charging, we consider only a portion of vehicles and roads to be electric vehicles and electrified links, and examine how the system performance changes with different levels of market penetration of electric vehicles and degrees of road electrification. Considering the second-best scenario is more practical, we conduct sensitive analyses upon it, and use the same network setting as above.

5.2.1. Market penetration of electric vehicles

In this section, a portion of vehicles are considered to be electric vehicles while the rest are regular gasoline-powered vehicles, whose travel costs include travel time and gasoline expense. In this situation, the traffic flow pattern will be in user equilibrium for each class of vehicles, i.e., multi-class user equilibrium. Subsequently, IPD needs to be extended to a multi-class integrated design problem or IPD-MC, which is reported in the Appendix A of this paper.

We split the OD demand of each pair into the electric and regular vehicles as per the market penetration. The gas consumption rate and price were assumed to be 0.04 gal/mi and $3.79/gal. The other inputs are the same as those in Tables 1–4. IPD-MC was solved with varying the market penetration. Fig. 3 reports the resulting social cost.

It can be observed that, as the market penetration of the electric vehicles increases, the social cost decreases. This can be attributed to two factors: one is the cost of energy consumption of electric vehicles is much cheaper than that of regular vehicles, and the other is that a higher percentage of electric vehicles makes our integrated pricing instrument more effective. It is also interesting to observe that the curve is approximately linear, and the benefit of electrifying vehicles appears not marginally diminishing. However, this observation may not necessarily hold in other networks.

Table 8
Power injection and locational marginal price at each bus.

<table>
<thead>
<tr>
<th>At node</th>
<th>Power injection (MW)</th>
<th>Locational marginal price ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>80.03</td>
<td>19.87</td>
</tr>
<tr>
<td>11</td>
<td>80.03</td>
<td>19.87</td>
</tr>
<tr>
<td>12</td>
<td>210.00</td>
<td>19.87</td>
</tr>
<tr>
<td>13</td>
<td>0.00</td>
<td>19.87</td>
</tr>
<tr>
<td>14</td>
<td>210.00</td>
<td>19.87</td>
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<tr>
<td>15</td>
<td>80.03</td>
<td>19.87</td>
</tr>
<tr>
<td>16</td>
<td>0.00</td>
<td>19.87</td>
</tr>
<tr>
<td>17</td>
<td>0.00</td>
<td>19.87</td>
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<td>18</td>
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<td>16.60</td>
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<td>19</td>
<td>0.00</td>
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<td>20</td>
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<td>21.64</td>
</tr>
<tr>
<td>21</td>
<td>300.00</td>
<td>21.64</td>
</tr>
</tbody>
</table>

Fig. 3. Changes in social cost with respect to market penetration of electric vehicles.
5.2.2. Level of road electrification

Considering electrifying the whole network will not be practical in the near future, we conducted another sensitive analysis on the level of road electrification. In order to formulate this problem, we assume that when traveling on regular unelectrified roads, electric vehicles essentially function as hybrid electric vehicles and consume gasoline. Under this assumption, the travel cost of electric vehicles includes travel time, electricity and gas expenses. Once again, IPD needs to be reformulated to incorporate this relaxation and the IPD with partial electrification or IPD-PE is also reported in the Appendix.

To demonstrate the impact of the level of electrification, we selected one of the most congested paths, i.e., 1–6–8–4 between OD pair 1–4 and solved IPD-PE under three different scenarios, i.e., link 1–6 is electrified, both links 1–6 and 6–8 are electrified, and all the three links are electrified. Moreover, in each scenario, the rest roads are regular and unelectrified. The corresponding social costs are presented in Table 9.

Table 9
Changes in social cost with respect to level of electrification.

<table>
<thead>
<tr>
<th>Electrified links</th>
<th>Social cost ($)</th>
</tr>
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<tbody>
<tr>
<td>1–6</td>
<td>728,780</td>
</tr>
<tr>
<td>1–6, 6–8</td>
<td>625,953</td>
</tr>
<tr>
<td>1–6, 6–8, 8–4</td>
<td>558,907</td>
</tr>
</tbody>
</table>

Fig. 4. A medium-size coupled transportation and power network.

5.2.2. Level of road electrification

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In order to demonstrate the potential of the manifold suboptimization algorithm for solving more realistic problems, we solved IPD using the coupled network that we created based on the topology of the Sioux Falls road network (24 nodes, 76 links and 182 OD pairs) and a subset of the IEEE 118-bus system (http://motor.ece.iit.edu/Data). Fig. 4 shows the topology of the coupled network. Note that although the road network has the same topology as the Sioux Falls network, it has been scaled up to represent a regional freeway network.

The IPD problem for this network contains 48,562 variables and 76,339 constraints, of which 13,920 are complementarities. It took a 3.06 GHz Dell Optiplex 380 computer with 3.00 GB of RAM 98 s to solve the problem. The optimal social cost is $214,685 and the electricity retail price on each link is shown in Table 10. Moreover, we conducted the same sensitive analyses as Section 5.2 and made similar observations.

6. Conclusions

Envisioning that wireless charging technologies will transform roads into pervasive charging infrastructure, we have proposed two models to explore integrated pricing of roads and electricity in transportation and power networks coupled by electric vehicles. In the scenario where a government agency manages both transportation and power systems, the first-best price of electricity at a location is found to be the marginal cost of providing electricity at the location, i.e., locational marginal price, while the first-best road tolls will internalize the marginal external travel time and the marginal external energy cost. The tolls will force individual vehicles to minimize their impacts on other vehicles and the operations of the power grid when choosing routes from their origins to destinations. In a second-best condition, the government agency can participate in a competitive wholesale power market as a buyer and then adjust the retail price of electricity to regulate route choices of

<table>
<thead>
<tr>
<th>Link</th>
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<tbody>
<tr>
<td>1–2</td>
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<td>1–3</td>
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<td>3–4</td>
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### Table 10 (continued)

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<tr>
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</tr>
<tr>
<td>18–7</td>
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</tr>
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5.3. Solving realistic networks

In order to demonstrate the potential of the manifold suboptimization algorithm for solving more realistic problems, we solved IPD using the coupled network that we created based on the topology of the Sioux Falls road network (24 nodes, 76 links and 182 OD pairs) and a subset of the IEEE 118-bus system (http://motor.ece.iit.edu/Data). Fig. 4 shows the topology of the coupled network. Note that although the road network has the same topology as the Sioux Falls network, it has been scaled up to represent a regional freeway network.

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electric vehicles. The optimal second-best pricing strategy can be obtained by solving a mathematical program with complementarity constraints with a manifold suboptimization algorithm. The strategy has been demonstrated to yield substantial cost reduction in a numerical example. Our sensitive analyses suggest that promoting electric vehicles and electrifying roads will both reduce the expenses of energy consumption and provide the government agency more flexibility of applying the integrated pricing strategy to regulate traffic flow. Both effects can help reduce the social cost associated with the transportation system.

One possible extension to the proposed models in this paper would be to consider time-varying demands of travel and electricity. Another extension is to incorporate vehicle-to-grid (V2G) technologies (e.g., Kempton and Letendre, 1997; Kempton and Tomić, 2005). More specifically, the government agency can purchase electricity not only in the power market, but also from electric vehicles that discharge their stored electricity to the power grid via V2G technologies. Although both extensions appear to be major undertakings since they integrate a dynamic traffic assignment model (e.g., Peeta and Ziliaskopoulos, 2001; Ben-Akiva et al., 2012) with a time-dependent power flow model (e.g., Wang et al., 2010) and involve more complex market mechanism design, the concepts of the first-and second-best pricing presented in this paper still apply.

Acknowledgments

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Appendix A

This appendix presents the formulations of the integrated design problem with multiple classes (IPD-MC) and with partial electrification (IPD-PE).

For IPD-MC, let we use the subscripts “re” and “ev” to represent variables associated with regular and electric vehicles respectively.

\[
\begin{align*}
\min_{(p_a, p_k, x_{rs}^{e_a}, x_{rs}^{r_e}, x_{rs}^{e_d}, x_{rs}^{r_d})} & \quad \sum_{a} f_{a}(v_a) u_a + \sum_{(k,a) \in C} v_{a \leftrightarrow} x_{a \leftrightarrow} d_{a \leftrightarrow} e_{a} + \sum_{a} v_{a, re} \hat{e}_{r} p_{g} d_{a} \\
\text{s.t.} & \quad (5) - (10) \text{ and } (13) - (22) \\
\nu_{a, ev} &= \sum_{rs} x_{a \leftrightarrow} x_{a \leftrightarrow} \forall a \in A \\
\nu_{a, re} &= \sum_{rs} x_{a \leftrightarrow} x_{a \leftrightarrow} \forall a \in A \\
\Delta x_{re}^{e_a} &= E^{e_a} q_{re}^{e_a} \forall r, s \\
\Delta x_{ev}^{f_a} &= E^{f_a} q_{ev}^{f_a} \forall r, s \\
x_{a \leftrightarrow}^{e_a} x_{a \leftrightarrow}^{e_d} &\geq 0 \forall a, r, s \\
\nu_{a} &= \nu_{a, ev} + \nu_{a, re} \forall a \in A \\
g_k - l_k - \sum_{(k,a) \in C} \nu_{a, ev} d_{a} e_{a} - \sum_{km} f_{km} = 0 \forall k \\
\left(\omega_{a} + p_{g} d_{a} e_{a} + \rho_{a}^{e_{a}} \right) \cdot x_{a \leftrightarrow}^{e_{a}} &= 0 \forall a = (i,j) \in A, r, s \\
\omega_{a} + p_{g} d_{a} e_{a} - \rho_{a}^{e_{a}} \geq 0 \forall a = (i,j) \in A, r, s \\
\left(\omega_{a} + p_{g} d_{a} e_{a} - \rho_{a}^{e_{a}} \right) \cdot x_{a \leftrightarrow}^{e_{a}} &= 0 \forall a = (i,j) \in A, r, s \\
\omega_{a} + p_{g} d_{a} e_{a} - \rho_{a}^{e_{a}} \geq 0 \forall a = (i,j) \in A, r, s \\
\sum_{a} v_{a, ev} p_{g} d_{a} e_{a} &\geq \sum_{(k,a) \in C} v_{a, re} \hat{e}_{r} p_{g} d_{a}
\end{align*}
\]

where \( p_{g} \) and \( \hat{e}_{r} \) represent the given gasoline price and gasoline consumption rate.

In the above, the objective function is to minimize the social cost, including the gasoline consumption. Constraints (46)-(51) define the set of feasible multi-class flow distributions in the transportation network. Constrains (53)-(56) ensure the multi-class user equilibrium. Constraint (57) is the same as (25).

For IPD-PE, we denote the sets of electrified and unelectrified roads as \( A_{ev} \) and \( A_{re} \). The formulation is as follows:
References


Feng, W., Filippuzzi, M., 2013. An economic and technological analysis of the key factors affecting the competitiveness of electric commercial vehicles: a case study from the USA market. Transportation Research Part C 26, 135–145.


