06a Consolidation and Settlement in Clay

Ref: Principles of Geotechnical Engineering, Braja M. Das, 1994
Dr. D. Bloomquist Notes

Topics:
Effective Stress
Consolidation and Compression
Preconsolidation
Settlement (in Clay)
Calculation of Primary Consolidation Settlement
Calculation of Secondary Consolidation Settlement
Summary
Time Rate of Consolidation

Effective Stress
The effective vertical stress ($\sigma_\text{v}$) at point $A$ below the soil surface is equal to the stress carried by the soil solids at their points of contact (i.e. the soil skeleton). It is found by subtracting the stress carried by the water in the continuous void spaces (i.e. pore water pressure, $u$) from the total stress of the weight of the overlying material ($\sigma_\text{w}$).

\[
\sigma_\text{v} = \sum \gamma_i z_i \rightarrow \sigma_\text{v} = \gamma_\text{w} H + \gamma_\text{sat} H_A \\
u = \gamma_\text{w} z_w \rightarrow u = \gamma_\text{w} (H + H_A) \\
\sigma'_\text{v} = \sigma_\text{v} - u \rightarrow \\
\sigma'_\text{v} = \gamma_\text{w} H + \gamma_\text{sat} H_A - \gamma_\text{w} (H + H_A) \\
= (\gamma_\text{sat} - \gamma_\text{w}) H_A = \gamma' H_A
\]

where $\gamma_\text{sat} = \frac{W_\text{s} + W_\text{w}}{V_{\text{total}}}$, $\gamma' = \gamma_\text{sat} - \gamma_\text{w} = \frac{W_\text{s}}{V_{\text{total}}}$

Consolidation
The reduction of bulk soil volume under loading due to flow of pore water. For saturated soils, any increment of loading ($\Delta\sigma$, called surcharge) will be initially taken up by the pore pressure and result in consolidation until a new equilibrium is reached where the soil solids (or skeleton) takes up the added load.

surcharge: $\Delta\sigma = \Delta\sigma' + \Delta u$
For cohesive soils: 
\[ \Delta \sigma = \Delta \sigma' \] 
\[ \Delta \sigma = \Delta \sigma' \]

For non-cohesive soils: water drains faster and the load is transferred immediately \( \rightarrow \) "consolidation" does not occur in non-cohesive soils; in non-cohesive soils this process is called "compression".

Preconsolidation Condition
1. **normally consolidated** - present effective overburden pressure = maximum pressure the soil has been subjected to in the past \( (p_c) \)

2. **overconsolidated** - present effective overburden pressure < maximum pressure the soil has been subjected to in the past \( (p_c) \) \( \ldots \) i.e. a load has been removed

Recall: \( e \equiv \text{void ratio}, \quad e = \frac{V_{\text{voids}}}{V_{\text{solids}}} = \frac{V_v}{V_s} \)

When a soil is loaded, it consolidates over the virgin consolidation curve (lefthand plot). If the load is removed (or partially removed) it will rebound non-linearly over a less steep curve (righthand plot). Adding a new load (i.e. due to construction) will cause the soil to consolidate over the less steep curve until it reaches the maximum pressure load from the past \( (p_c \text{ or } \sigma_c') \). Then it will follow the steep curve again. If the new load is less than \( p_c \) settlement will be small (following the shallow curve).
Preconsolidation pressure determination
(Casagrande, 1936)
1. $e$-$\log p$ is established by lab testing
2. determine point $a$ at which $e$-$\log p$ has minimum radius of curvature
3. Draw horizontal line from $a$ (line $ab$)
4. Draw tangent to curve at $a$ (line $ac$)
5. Draw line $ad$ to bisect angle $bac$
6. Project the straight-line portion of $gh$ back to intersect $ad$ at $f$
7. Abscissa of point $f$ is the preconsolidation pressure, $p_c$

Compression/Consolidation of soil layers due to stress increase by construction of foundations or other loads. Compression is caused by:
1. Deformation of soil particles
2. Relocation of soil particles
3. Expulsion of water or air from void spaces

Settlement ($\rho$)

1. Immediate settlement ($\rho_i$) - elastic deformation of dry soil and moist and saturated soils without change to moisture content
   a. due to high permeability, pore pressure in clays support the entire added load and no immediate settlement occurs
   b. generally, due to the construction process, immediate settlement is not important
2. Primary consolidation settlement ($\rho_c$) - volume change in saturated cohesive soils because of the expulsion of water from void spaces
   a. high permeability of sandy, cohesionless soils result in near immediate drainage due to the increase in pore water pressure and no primary (or secondary) "consolidation" settlement occurs, only immediate settlement
3. Secondary consolidation settlement (creep) ($\rho_s$) - plastic adjustment of soil fabric in cohesive soils
Calculation of Primary Consolidation Settlement

Given: Saturated clay soil layer
- thickness = \( H \)
- cross-sectional area = \( A \)
- existing overburden pressure = \( p_o \)
- increase in pressure = \( \Delta p \)

Find: resulting primary consolidation settlement \( \rho_c \)

Change in volume is \( \Delta V = -\rho_c A \), change in volume is equal to the change in volume of the voids (definition of settlement) and by the definition of the void ratio:

\[
\rho_c A = -\Delta V = -\Delta V_v = -\Delta e V_s = -(e_f - e_0)(H_v - H_v^0)A
\]

\[
\text{initial void ratio:} \\
e_0 = \frac{V_{v0}}{V_s} = \frac{H_{v0}}{H_s} \\
V = HA
\]

\[
\text{final void ratio:} \\
e_f = \frac{V_{vf}}{V_s} = \frac{H_{vf}}{H_s} \\
V = HA
\]

\[
\rho_c A = -\Delta V = -\Delta V_v = -(H_v - H_v^0)A = -\Delta e V_s, \text{ where } \Delta e = e_f - e_0 = \frac{H_{vf} - H_{v0}}{H_s}
\]

\[
\rho_c = -\Delta e H_s = -(H_v - H_v^0) = -\Delta H
\]

Using the initial void ratio and total volume \((e_0 \text{ and } V)\) gives \( V_s = \frac{V}{1 + e_0} = \frac{AH}{1 + e_0} = AH_s \)

Combining and rearranging gives \( \rho_c = -\Delta H = -\Delta e H_s = -\frac{\Delta e}{1 + e_0}H \)
Normally Consolidated Soil (Clay)

Compression index $C_c = \text{slope of the } e\text{-log } p \text{ curve:}$

\[
C_c = \frac{-(e_f - e_0)}{\log(p_f) - \log(p_0)} = \frac{-\Delta e}{\log(p_f/p_0)} = \frac{-\Delta e}{\log[(\sigma_0' + \Delta\sigma_v')/\sigma_0']}
\]

\[
\rho_c = \frac{-\Delta e}{1 + e_0} H = \frac{C_c H}{1 + e_0} \log \left( \frac{\sigma_0' + \Delta\sigma_v'}{\sigma_0'} \right)
\]

- For thick clay, it is more accurate to divide $n$ multiple layers

\[
\rho_c = \sum_{i=1}^{n} \frac{C_{cl} H_i}{1 + e_{oi}} \log \left( \frac{\sigma_{oi}' + \Delta\sigma_{oi}'}{\sigma_{oi}'} \right),
\]

shear stress is computed at center of sub-layers

- Consider depth to $2B$ for square foundation (BxB) or $4B$ for strip foundations (BxL), $B$ is the width (below this depth, the load has dissipated and is zero)

Compression index determination

1. Graphically from laboratory $e\text{-log } p$ plot, use "virgin compression curve" (i.e. straight line portion of the curve)

2. Rendon-Herrero (1983) $C_c = 0.141G_s^{1.2} \left( \frac{1 + e_o}{G_s} \right)^{2.38}$

3. Nagaraj and Murty (1985) $C_c = 0.2343 \left[ \frac{LL(\%)}{100} \right] G_s$

Over-Consolidated Soil (clay)

**Highly over-consolidated**: load is less than maximum past load, $\sigma_0' + \Delta\sigma_v' \leq \sigma_c'$. 

Recompression index $C_R = \text{slope of the } e\text{-log } p \text{ rebound/reloaded curve:}$

\[
\rho_c = \frac{-\Delta e}{1 + e_0} H = \frac{C_R H}{1 + e_0} \log \left( \frac{\sigma_0' + \Delta\sigma_v'}{\sigma_0'} \right), \text{ typically } C_R \text{ is } 10\text{-}20\% \text{ of } C_c
\]
Lightly over-consolidated: load is greater than maximum past load,

\[ \sigma_0' + \Delta \sigma_v' > \sigma_c' \]

In general:

- For thick clay, divide into \( n \) multiple layers & use appropriate equation for each layer (shear stress is computed at center of sub-layers)

\[ \rho_c = \sum_{i=1}^{n} \rho_{ci} \]

- Consider depth to 2\( B \) for square foundation (BxB) or 4\( B \) for strip foundations (BxL), \( B \) is the width (below this depth, the load has dissipated and is zero)

- most marine soils are overconsolidated - sedimentation increases the surcharge on the soil, but subsequent erosion removes much of the load

Calculation of Secondary Consolidation Settlement

- Secondary consolidation is the portion of time-dependent settlement that occurs at essentially constant effective stress.
- The rate of secondary consolidation is not dependent on the flow of water or on clay layer thickness, and is relatively constant for normal engineering stress increases.
- Secondary consolidation occurs slowly with a continually decreasing rate.
- The secondary consolidation portion of the consolidation curve is approximately linear on an e-log time plot.
- Secondary consolidation usually estimated from a lab consolidation test.
- The exact cause of secondary consolidation is unknown, but is possibly a readjustment of the double water layer surrounding the clay particles.
- Secondary consolidation may be more important than primary consolidation for organic and highly compressible inorganic clays.
The ratio of secondary to primary consolidation increases as the ratio of stress increment to initial stress decreases: i.e. watch out for small stress increases in thick clay layers.

\[ \frac{\rho_s}{\rho_c} \uparrow \text{as} \frac{\Delta \sigma_v}{\sigma_0} \downarrow \]

"Secondary (compression) settlement is more important in organic and highly compressible inorganic soils. In overconsolidated inorganic clays, the secondary consolidation index is very small and of less practical importance."

Braja M. Das  
**Principles of Geotechnical Engineering**

Secondary Compression Settlement is generally only a concern for highly organic material (e.g. peat)

\[ C'_{\alpha} = \frac{C_{\alpha}}{1+e_0} \]

Note: \( C'_{\alpha} \) found by Mesri to have correlation with water content, can replace \( C_{\alpha}/(1+e_0) \) with \( C'_{\alpha} \) in above equation for \( \rho_s \) (see Das Fig. 8.19)

**Uniform \( C'_{\alpha}/C_c \) ratio (USACE)**

- Inorganic: 0.025 - 0.065
- Clay: 0.025 - 0.085
- Silt: 0.030 - 0.075
- Peat: 0.030 - 0.085
SUMMARY OF SETTLEMENT CALCULATIONS (Clays)

- **Define Initial Stresses:**
  - Total Stress, Pore Water Pressure, Effective Stress
  - Must define the state of stress prior to loading
  - The behavior of soils is governed by effective stress (thank you Karl Terzaghi)

- **Define Stress Change Due to Load:**
  - Load is applied at a point or pressure applied over an area
  - Model soil as linear-elastic, homogenous and isotropic
  - Boussinesq solutions for various shaped loads (influence factor x load)
  - FEM solutions possible but require extensive soil property characterization and knowledge of field variability (i.e. after assumptions may not be any better)

- **Define Settlement Due to Load, CLAYS:**
  - **Immediate, \( \rho_i \):**
    - Small movement, occurs quickly
    - Movement due to distortion (zero volume change)
    - Uses Boussinesq stress solutions
    - Model soil as linear-elastic and isotropic
    - Affected by shape and stiffness of footing, \( \rho_i = \text{influence factor} \times \text{load} \)
  - **Consolidation, \( \rho_C \):**
    - Movement can be large, occurs over time, due to volume (void ratio, \( e \)) change
    - Analyses assume saturation and are usually 1-D
    - Calculate total stress change (\( \Delta \sigma_V \)) using Boussinesq methods
    - At \( t=0 \):
      - \( \Delta u = \Delta \sigma_V \) \( \Rightarrow \) water supports load as excess pore water pressure
      - \( \Delta \sigma'_V = 0 \) \( \Rightarrow \) effective stress has not changed (yet)
    - As water seeps out \( \Rightarrow \Delta u \) and \( e \) decrease, \( \Delta \sigma'_V \) increases, and settlement occurs
    - At \( t=\infty \):
      - \( \Delta u = 0 \) \( \Rightarrow \) excess pore pressure decreases to zero
      - \( \Delta \sigma'_V = \Delta \sigma_V \) \( \Rightarrow \) effective stress now supports load
    - Rate of load transfer (and settlement) characterized by theoretical equation (thank you again Karl Terzaghi) which has numerical solution using dimensionless time factor \( T \), depth factor \( Z \), and coefficient of consolidation \( c_V \)
    - Soil remembers max. past stress to which it has already consolidated
    - OCR = Over-consolidation Ratio = \( (\sigma'_C / \sigma'_V) \)
    - If NC soil (OCR=1) then virgin compression: use \( C_C \) to find \( \Delta e \) & \( \rho_C \)
    - If OC soil (OCR>1) then recompression up to \( \sigma'_C \): use \( C_R \) to find \( \Delta e \) & \( \rho_C \), and virgin compression above \( \sigma'_C \): use \( C_C \) to find \( \Delta e \) & \( \rho_C \)
    - 1D (Oedometer) lab measurement of consolidation properties (\( C_R, C_C, c_V \))
  - **Secondary Consolidation, \( \rho_S \):**
    - Movement usually small (except for organic & highly compressible clays)
    - Due to volume (void ratio) change at constant effective stress
    - Get secondary compression index (\( C_S \)) from lab, use to find \( \rho_S \) for any \( \Delta t \) after \( \rho_C \)
**Time Rate of Consolidation**

**Derivation assumptions**

1. Homogeneous clay-water system
2. Saturated
3. Water and soil grains are incompressible
4. Flow of water is unidirectional and in the direction of consolidation
5. Darcy's law assumed: \( v = k \)
   
   where \( v \) = discharge velocity, \( k \) = coeff of permeability, \( i \) = hydraulic gradient, \( i = \Delta h/L \)

\[
\frac{\partial v_z}{\partial z} \text{dxdydz} = -\frac{\partial V}{\partial t}
\]

Darcy's law gives

\[
v_z = -k \frac{\partial h}{\partial z} = -\frac{k}{\gamma_w} \frac{\partial u}{\partial z}
\]

since \( u = \gamma_w h \)

combining gives

\[
\frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = \frac{1}{\text{dxdydz}} \frac{\partial V}{\partial t}
\]

during settlement

\[
\frac{\partial V}{\partial t} = \frac{\partial V_v}{\partial t} = \frac{\partial (V_s + eV_s)}{\partial t} = \frac{\text{dxdydz} \partial e}{1 + e_o} \frac{\partial t}{\partial t}, \text{ since } \frac{\partial V_s}{\partial t} = 0 \text{ and }
\]

\[
V_s = \frac{V}{1 + e_o} = \frac{\text{dxdydz} \partial e}{1 + e_o} \Rightarrow \frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = \frac{1}{1 + e_o} \frac{\partial e}{\partial t}
\]

assume that the decrease in void ratio is proportional to the increase in effective stress (or the decrease in pore pressure) \( \partial e = -a_v \partial u \), \( a_v = \text{coeff. of compressibility} \)
define the coeff. of volume compressibility ($m_v$)

$$m_v = \frac{a_v}{1+e_o} \to k \frac{\partial^2 u}{\partial z^2} = m_v \frac{\partial u}{\partial t},$$

define coeff. of consolidation ($c_v$)

$$c_v = \frac{k}{\gamma_w} m_v = \frac{k(1+e_o)}{\gamma_w a_v} \to \frac{\partial^2 u}{\partial z^2} = c_v \frac{\partial u}{\partial t}$$ (Terzaghi's 1D consolidation theory)

solving gives a time factor $T_v = \frac{c_v t}{(H/N)^2}$

2 drainage paths: above & below

N = 2

1 drainage path: bottom layer is impermeable

N = 1

estimate $m_v$ from $e$-$\log p$ plot at appropriate pressures, $m_v = \frac{1}{1+e_{av}} \frac{\Delta e}{\Delta p}$, $e_{av} = \frac{e_1 + e_2}{2}$

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Sivaram & Swamee (1977) empirical relationship for $U$ (degree of settlement) from 0-100%

$$\frac{U\%}{100} = \sqrt{\frac{4T_v}{\pi}} \left[1 + \left(\frac{4T_v}{\pi}\right)^2\right]^{-0.179}$$

and $T_v = \left(\frac{\pi}{4}\right) \left[\frac{U\%}{100}\right]^2 \left[1 - \left(\frac{U\%}{100}\right)^{5.6}\right]^{-0.357}$