
Solve the one way wave equation, $\frac{\partial u}{\partial t} + C \frac{\partial u}{\partial x} = 0$, using second and fourth order explicit methods.

A. Use second order Leap Frog time difference and central spatial differences on the domain, 0 < x < 10 m, on the time interval, 0 < t < 6 s, with $C = 1 \text{ m s}^{-1}$, with initial conditions $u(x,0) = 3e^{-5(x-2)^2}$. Let $\Delta x = 0.1 \text{ m}$ (101 grid points).

The exact result should look like $u(x,6) = 3e^{-5(x-8)^2}$. Plot this together with your solution for $C_f = 0.1, 0.99, \text{ and } 1.2$. For boundary conditions use $u = 0$ at $x = 0, 10 \text{ m}$ at all times. For the first time step, use an Euler (1st order) time step.

B. Repeat using $\Delta x = 0.01$ and 0.5 m, with $C_f = 0.99$.

C. Compare the solutions for the L-F method at $t = 6 \text{ s}$ and $t = 6 - \Delta t \text{ s}$ for $C_f = 0.99$.

D. Repeat using 4th order central spatial differences with $\Delta x = 0.1$ and 0.01 m, $C_f = 0.5$ and 0.99 (at least one of these should be unstable).

E. Use 1st order Euler time step instead of L-F with $C_f = 0.5$, $\Delta x = 0.1 \text{ m}$, to see if an instability is present.

Plot the results (e.g., with Tecplot). See the example Code distributed by email.