Axial convergence fronts in a barotropic tidal inlet—sand shoal inlet, VA

Chunyan Li*

Skidaway Institute of Oceanography, 10 Ocean Science Circle, Savannah, GA 31411, USA

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Abstract

An axial convergence front system in Sand Shoal Inlet, VA, is analyzed. The fronts, which are parallel to the main channel and mostly within the channel, occur during different tidal stages within a 13-h observation period. A 25-ft boat is used to tow an acoustic Doppler current profiler to measure velocity profiles along an hour-glass shaped ship track. Salinity and temperature are measured using conductivity–temperature–depth sensors. A harmonic-statistic analysis is used to analyze the tide, tidal velocity, and mean velocity. The transverse convergence and divergence of velocity are calculated. The rms errors of the harmonic-statistic analysis of the elevation and velocity are about 0.28 m and 0.13 m/s (with a maximum velocity of over 2 m/s), respectively. On average, about 83%, 95%, and 70% of the variabilities of the elevation, longitudinal and transverse velocities, respectively, can be explained by the $M_2$ tidal and subtidal constituents. Strong transverse velocity convergences are identified by the analysis and are generally consistent with the observed front positions. The analysis shows that the front system is apparently generated by a combination of several mechanisms including (1) differential rotation of the tidal ellipses and spatial variations of the major axes of the tidal ellipses, owing to the strong bottom friction, and (2) a strong geometric convergence at the inlet. Density effect is found to be negligible and the planetary vorticity tilt effect is also unimportant because of a much higher relative vorticity. The observed front system is distinguished from the conventional estuarine axial convergence fronts which require a strong along channel density gradient. Therefore, in estuaries and coastal embayment where both tides and density gradients are important and where cross-channel bathymetry change is present, the frontal genesis can be a combination of several processes including the tidal convergence discussed here.

Keywords: Convergence fronts; Tides; Tidal inlets; Axial fronts; Acoustic Doppler current profiler (ADCP); Observations

1. Introduction

Fronts in coastal waters are caused by convergence of flows. In an estuarine environment, when there is a significant seaward freshwater discharge, a longitudinal density gradient is established. If at the same time, there is a cross channel variation of the longitudinal flow, this velocity shear will redistribute the density field to form a cross channel density gradient. Assuming the velocity is larger over the channel than the shoals (Fig. 1a), the resultant density over the channel during flood will be larger than that over the shoals (Fig. 1b) which will produce a (transverse) pressure gradient toward the channel on the surface and a pressure...

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*Fax: +1-912-598-2310.
E-mail address: chunyan@skio.peachnet.edu (C. Li).
gradient toward the shoals at the bottom. A convergence will thus tend to form on the surface over the channel with a compensating divergence occurring at the bottom of the channel. These are the so-called “axial convergence fronts” (Nunes and Simpson, 1985) or shear fronts (O’Donnell, 1993). During ebb, the pressure gradients will be reversed and the surface convergence will be replaced by a divergence. The assumption that the velocity over the channel is larger than that over the shoals is often valid in a shallow water environment. The strong bottom friction results in a smaller (larger) tidal velocity amplitude in shallower (deeper) water as demonstrated by two-dimensional (2D) (Li and Valle-Levinson, 1999) and three-dimensional (3D) analytical models (Li, 2001) and observations in, for example, San Francisco Bay (Cheng and Gartner, 1985), York River Estuary (Huzzey and Brubaker, 1988), Chesapeake Bay (Valle-Levinson and Lwiza, 1995), and James River Estuary (Li and Valle-Levinson, 1999). Theoretically, the fronts associated with this type of convergence flow require a longitudinal density gradient and occur only during flood tides (Nunes, 1982; Nunes and Simpson, 1985; Simpson and Turrell, 1986; Largier, 1992; Swift et al., 1996; Turrell et al., 1996). Although observations sometimes suggest that the fronts may also occur during late flood and/or early ebb, the processes have been attributed to the same mechanism (Sarabun, 1980; Huzzey and Brubaker, 1988).

A density gradient is not a necessary condition for axial convergence fronts in estuaries and tidal channels. Frictional tide and bathymetry can combine to generate convergences (Li, 2001) by either a differential rotation of tidal ellipses across depth changes (Fig. 2a) or a spatial variation of the major axis of the tidal ellipses (Fig. 2b) (Li and Valle-Levinson, 1999; Valle-Levinson et al., 2000). The differential rotation of tidal ellipses is caused by a frictional phase lag of tidal flows between deep water and shallow water. In general, the tidal phase in shallow water leads that in deep water as shown by observations (e.g. Sarabun, 1980) and analytic models (Li and Valle-Levinson, 1999; Li, 2001). As such, the velocity vectors across the channel of variable depth are generally not parallel to each other. The difference of angles of adjacent velocity vectors on the same cross section changes over the tidal cycle. Depending on the directions of rotation of the velocity vectors over time, a convergence or divergence can thus occur at different tidal stages (Fig. 2a). The differential rotation of tidal ellipses tends to generate axial convergence and divergence since in tidal channels

![Fig. 1. Mechanism of axial convergence fronts: interaction between longitudinal density gradient and longitudinal flow with lateral shear. Shown here are plane views of surface density and flow. (a) Hypothetical initial state with uniform cross channel density field and a cross channel shearch of surface velocity. (b) Resultant density field and convergent surface flow. The dotted lines are the density contours.](image1)

![Fig. 2. Mechanism of axial convergence fronts: tidal convergences. (a) Differential rotation of tidal ellipses across bathymetry changes. (b) Tilt of the major axis of the tidal ellipses caused by changes in bathymetry or the existence of lateral boundary.](image2)
and estuaries, depth contours are usually aligned in the longitudinal direction.

A spatial variation of the major axis of the tidal ellipses is a result of a variation in the ratio between the along channel and cross channel velocities. In general, a significant change in the cross channel velocity, which often occurs across bathymetry changes or near solid boundaries, will result in the change of this ratio. Usually, the change of this ratio varies along the channel and can be combined with the effect of the differential rotation of tidal ellipses to strengthen axial convergences and divergences. The difference between the differential rotation and the tilt of ellipses is that the latter does not require a phase difference in velocity between adjacent locations.

In a frictional flow condition, axial convergence can also be generated by a combination of Coriolis effect and depth variations through the vortex tilt mechanism and boundary effect (Mied et al., 2000). As shown in Fig. 3a, after a horizontal vorticity is generated by the frictional tilt of the vertical planetary vorticity (Pedlosky, 1987), the flows of the vortex tubes cancel each other in the interior. At the same time, the side boundaries on the banks will result in a net vortex such that, if the observer is facing the direction of the flow vector, a transverse flow to the right on the surface will form while a return flow to the left at the bottom will occur to form a clockwise circulation cell in northern hemisphere (Fig. 3b). In the southern hemisphere, a counter-clockwise circulation can be formed if the observer is facing the direction of the flow. This mechanism can generate axial convergence on the right bank of the flow (in the northern hemisphere). Obviously, the convergence position changes from flood to ebb (Handler et al., 2000).

In this paper, we examine an axial convergence front system observed in a tidal inlet where the density gradient is negligible and Coriolis effect is limited. High resolution divergence and convergence are calculated from velocity data obtained by a vessel towed ADCP. The flow convergences are used to explain the observed front system.

2. Study area and observations

Sand Shoal Inlet, VA (37.30°N, 75.78°W) is located at the ocean side of the Delmarva Peninsula (Fig. 4) as the deepest opening to the eastern shore lagoon, an area of great interests on ecology especially the recruitment of fish larvae (Brumbaugh, 1996). Tidal range is between 1 and 2 m, depending on the lunar phase. The coastal circulation in the Mid-Atlantic Bight (MAB) is alongshore and southward with velocities between 5 and 15 cm/s. Mean salinity of the offshore coastal water in the MAB area is about 32.5 psu. Wind in the area is predominantly from the northeast (late summer through early spring) and southwest (summer). Typical wind speed is 4–6 m/s except in the summer when the wind is usually weak. There has been little physical oceanographic studies in the area except some observations using a hand operated current meter at a single station by Brumbaugh (1996).

Our original objective of the study in the Sand Shoal Inlet was to determine basic hydrography and tidal current field in the area. The study was conducted with an intensive rapid survey using an ADCP and a CTD along an hour-glass shaped ship track (Fig. 4). The ship track was designed to resolve the spatial variabilities of flows. A
600 KHz RD Instruments broadband direct-reading ADCP was towed 5 m off one side of a 25 ft boat. The ADCP was mounted at a towed body surfing on the surface with the transducers looking downward. The ADCP was configured to sample with 0.5 m vertical bins and 30 s averaging intervals. The boat was traveling along the ship track at an average speed of 5 knots (2.5 m/s). The average horizontal resolution was thus about 75 m. A differential GPS was used for the navigation. The position and time from the GPS were saved with the ADCP velocity profile data for each ensemble. Since the maximum depth is about 23 m, the ADCP was able to operate in the “bottom tracking mode” throughout the survey. The temperature sensor on the ADCP provided a continuous record of surface water temperature. One CTD station was selected at midway of each of the two transects perpendicular to the main channel (Fig. 4). One CTD cast at each of the two stations was performed during each repetition along the track. A total of 16 repetitions were completed during the 13-h observations on May 12, 1999 from 7:00 AM to 8:00 PM local time. The date of observations was 2 days before spring tides in the area.

Weather condition during the whole observations was favorable. Wind was very weak and there were no significant waves in the study area during the day. In contrast, tidal current was very strong (more than 2 m/s at maximum flood and ebb) which suggests that local wind effect should be very limited. Water temperature was about 17.5°C in the morning and rose up to 20.5°C on the surface at noon and dropped back to 18.5°C in the evening before the end of the survey. Surface temperature showed maximum spatial variation of about 1.4°C. The vertical variation of temperature was less than 1°C throughout the tidal cycle. Temporal variation of salinity was between 30.2 and 30.6 psu. Horizontal difference of salinity was less than 0.1 psu and vertical difference of salinity was between 0.02 and 0.2 psu. Because of the small variation of salinity, density was mainly controlled by temperature and reached its minimum from 11:00 AM to 3:00 PM, even though during this time period salinity was at it maximum. As a result, density variation was small both vertically (±0.1 kg/m³ from the mean) and horizontally (±0.2 kg/m³ from the mean). In summary, during the observational period, the inlet was very well mixed both horizontally and vertically. The strong
barotropic tidal motion appeared to be the dominant component.

3. Axial convergence front system

Not long after the observations were commenced early in the morning when it was ebbing, we noticed some very strong axial fronts identifiable with lines of foams and debris at various locations on both sides of the inlet. The timing of each encounter of the axial front was hand recorded. By comparing the recorded timing with the ADCP records, which included both time and latitude and longitude, we were able to mark the positions of the observed fronts. Since the ADCP data was recorded every 30 s, and the reading and writing of time from GPS usually took less than 15 s, we estimated that the average error of front positions thus determined were about the distance within ±15 s of the boat time or roughly ±40 m, assuming we could ignore the GPS position error. Although efforts were made to record every front the boat crossed, we might have missed a relatively small number of them when we occasionally made notes or checked the operation of ADCP and when we changed crew at noon. With these limitations in mind, we have assembled as complete as possible about 40 positions of fronts during the 13-h survey. On average, we recorded 2–3 front positions during each of the 16 repetitions along the hour-glass shaped ship track.

Notice that these axial fronts were first observed during ebb and the CTD data showed no significant stratification either horizontally or vertically. It is therefore clear that these fronts were not caused by density gradient as shown in Fig. 1. At the same time, the fronts appeared on both sides of the channel, indicating that the planetary tilting mechanisms (Fig. 3), which would only produce a front on the right side facing the flow direction, could not be applied to explain them. Fig. 5 shows some examples of surface flow field and the observed front positions during different time periods. Most of these front positions appear to be located at where strong surface convergences were present. Fig. 6 shows all the front positions recorded during the 13-h observations. As in Fig. 5, the water depth contours were also shown in Fig. 6 which indicate that most of the fronts were limited within the channel and very few of them appeared on the shallow water. Interestingly, this is somewhat similar to the fronts observed in the Delaware Bay (Klemas and Polis, 1977a,b) and York River Estuary (Huzzey and Brubaker, 1988), though these earlier observations had significant density gradients.

The uniqueness of these observations is that we not only recorded the front positions but also collected the velocity profiles throughout the observations over one tidal cycle. By analyzing the flow field, we may identify surface convergences and compare with the front positions. The flow convergences can be quantitatively determined and the cause of these fronts analyzed. Therefore, in the data analysis presented below, we have two main objectives: (1) determine the tidal characteristics of the flow and the convergences so that the cause of the fronts may be determined, and (2) quantify the magnitude of these convergences and compare with the observed front positions. The maximum convergence zones should be consistent with the observed front positions.

4. Data analysis

Technically, to achieve our objectives outlined above, we will characterize the tide by analyzing (1) the magnitude of tidal elevation and velocity, (2) the phase difference between elevation and velocity and the spatial distribution of velocity phase, (3) the strength of friction, (4) spatial distribution of tidal ellipses and mean flow, (5) the strength of velocity shear, and (6) the strength of velocity convergence. The analysis involves several techniques. The core of which is a harmonic-statistic analysis based on Li et al. (2000), which is useful for interpreting both time series and space–time series data such as those obtained on a moving vessel. In Li et al. (2000) the harmonic-statistic analysis was applied to a single transect across the Chesapeake Bay entrance for water depth measured by a vessel-towed ADCP to extract information about the tidal elevation and its spatial distribution. In contrast, here we apply...
the harmonic-statistic method to both single trans-
sects and to a rectangular grid system covering the
study area for both tidal elevation and tidal velocity.

To determine the amplitude and phase of the
tidal elevation and tidal velocity as functions of the
horizontal and vertical positions, we first grid a
rectangular area covering the ship track (Fig. 7a)
and then apply the harmonic-statistic method for
each variable within each grid. The harmonic-
statistic method for each variable (depth or
elevation, and velocity components) within each
grid is described as the following.

We assume that a time-dependent variable
denoted by \( v \) within any grid can be expressed as

\[
v = a_0 + \sum_{j=1}^{M} \left[ z_j \cos(\omega_j t) + \beta_j \sin(\omega_j t) \right]
\]  

(1)

in which \( z_j \) and \( \beta_j \) (\( j = 0, 1, 2, \ldots, M \)) are harmonic
constants, \( \omega_j \) is the \( j \)'th tidal frequency, \( t \) is time,
and $M$ is the total number of tidal frequencies selected. In a matrix form, Eq. (1) is simply

$$V = Ax$$

in which $V$ is the variable vector for all $v$ values measured at different times in a given grid within the ship track covered area, $A$ is a matrix, and $x$ is a vector of the harmonic constants, i.e.

$$V = (v_1, v_2, \ldots, v_N)^T,$$  

$$x = (a_0, a_1, a_2, \ldots, a_{2M})^T, \quad x_{2k-1} = a_k,$$

$$x_{2k} = \beta_k, \quad k = 1, 2, \ldots, M,$$  

where $T$ denotes the transpose of the vector, and

$$A = \begin{pmatrix}
1 & a_{1,1} & a_{1,2} & \cdots & a_{1,2M} \\
1 & a_{2,1} & a_{2,2} & \cdots & a_{2,2M} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & a_{N,1} & a_{N,2} & \cdots & a_{N,2M}
\end{pmatrix},$$

where $N$ is the total number of observations and

$$a_{i,2k-1} = \cos(\omega_k t_i), \quad a_{i,2k} = \sin(\omega_k t_i),$$

$i = 1, 2, \ldots, N, \quad k = 1, 2, \ldots, M.$

Since Eq. (2) is usually ill-posed or over-determined ($N \gg M$), we choose the least-squares method to estimate the harmonic constants (the vector $x$). It can be readily shown that the best statistical estimate of $x$ is

$$\hat{x} = (A^T A)^{-1} A^T V.$$  

The error or the residual sum of squares is

$$R_{SS} = (V - A\hat{x})^T (V - A\hat{x})$$

and the standard deviation (or the rms error) of the fitting and the coefficient of determination are, respectively,

$$\sigma = \sqrt{\frac{R_{SS}}{N - (2M + 1)}}$$

and

$$R^2 = 1 - \frac{R_{SS}}{R_{VV}},$$

in which $R_{VV}$ is

$$R_{VV} = \sum_{i=1}^{N} \left( v_i - \sum_{j=1}^{N} \frac{v_j}{N} \right)^2.$$  

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Fig. 6. Position of fronts during different tidal stages. The filled color contours show the water depth in meters.
Eq. (10) is the fraction of variability explained by the tidal constituents plus the mean which is 1 minus the unexplained variability $R_{SS}/R_{VV}$. The core of this analysis, Eqs. (7) and (8), is in matrix forms and thus generic and concise computer codes can be readily designed using the Interactive Data Language (IDL) or MATLAB. In the following section we present the results of the analysis using Eqs. (7), (9), and (10) for the water depth and current velocity measured by the ADCP within each grid. In the analysis, we choose the $M_2$ tidal frequency only owing to the limited length (13 h) of the record.

Fig. 7. Analysis of tidal elevation from the water depth measured by the ADCP. (a) Defined grids covering the ship track for the harmonic analysis of water depth variations within each grid. (b) The elevation time series at all the grids and the harmonic fit.
5. Discussion of results

5.1. Tidal elevation

As a primary tool to measure velocity profiles, a vessel-towed ADCP can also be used to infer amplitude and phase of tidal elevation as well as their spatial distributions, providing a convenient way to picture the tidal characteristics more completely (Li et al., 2000). This provides an approach to correct the vertical coordinate affected by the moving surface due to tidal oscillations to obtain reliable subtidal velocity profiles. We now apply the technique of last section to the present study. Since the study area has only a horizontal scale of about 1 km, we ignore the spatial differences of tidal elevation and treat the elevation as having the same amplitude and phase at any position within the area.

To extract tide, we first mask the study area with 90 m grids (Fig. 7a). The smaller grid length along the north–south direction is simply aimed at a finer resolution to avoid a larger error of estimate because of larger bottom slopes in this (cross channel) direction. The larger bottom slopes result in larger variability in depth within the same grid. As concluded in Li et al., the error of estimate depends on the GPS error, ship speed, and bottom slope. The optimal grid size is comparable in magnitude to the horizontal resolution, or the length the ship travels within each ensemble, which is 75 m in our case. Here, our average grid scale is also 75 m $(90 + 60)/2$.

We then group the local water depth values within each grid as a single time series. Excluding those grids with fewer than five data points, we apply the traditional harmonic analysis using the $M_2$ tidal frequency and the zero frequency (mean depth) to each grid. By subtracting the local mean depth from the original data, we obtain a time series of elevation for each grid. Since we ignore the spatial difference in elevation, we then treat elevation from all grids as a single time series. The harmonic-statistic analysis is then applied to this time series to obtain the amplitude, phase, and the statistics of the estimate (Fig. 7b). The calculated tidal amplitude is 0.87 m and about 83% of the depth variation can be explained by the tidal motion. The mean error of estimation is about 0.28 m. As discussed in Li et al. (2000), the majority of the error is a result of the vessel moving over large bottom slopes.

5.2. Tidal velocity

The velocity data are also grouped within each grid before the harmonic-statistic analysis is applied. Since velocity is also dependent on vertical position, the analysis is performed for both depth averaged velocity and velocity at different depths. Fig. 8 provides some typical examples of the surface velocity and the harmonic fits within different grids. Shown in the figure are also the fits of tidal elevation (the dotted lines). The phase difference between tidal elevation and velocity is between 90° and 100°, indicating that tide is approximately a standing wave. This explains the unusually high tidal velocity ($\approx 2$ m/s).

To better visualize the spatial distribution of tidal velocity and its evolution over time, we also apply the harmonic-statistic analysis to the velocity on the four transects. A different grid system is used for this purpose. First, we define four transects with a finite width centered on the dotted lines of Fig. 6. For convenience, we name the transects represented by lines CD, AB, AD, and CB the East, West, East–West, and Diagonal Transects, respectively. The origin of the transect coordinates is either A or C. In actual calculations, we have extended the Diagonal Transect a little further to include the shallowest water at the southeastern corner of the study area. Because of the strong currents in the inlet, it was difficult for the boat operator to follow the planned route accurately. The CTD casts on the East and West transects caused large deviations of ship track from the planned route (Fig. 7a). To include as much data points as possible, we use a 100 m width for each transect. The finite-width transects are then divided into segments of 75 m, to be consistent with the average horizontal resolution. The selection of 100 m instead of 75 m for the width is simply to include as much data as possible, yet keep the quality of the analysis. A too large width would result in too much scatter of
Fig. 8. Examples of the along channel tidal velocity (cm/s) within different grids, the harmonic fit (solid lines) and the tidal elevation (cm) obtained in Fig. 7 (dotted lines). The vertical axis of the panels are for both velocity and elevation with different units as indicated.
data points whereas a too small width would result in lose of information. We have also tried different widths (including 75 m), and the results are similar with only a few percent of difference for all variables.

The amplitude, phase, and mean of the along channel and across channel velocities within each segment are obtained and the result is then used to reconstruct the evolution of surface velocity on the distance–time plane (Figs. 9 and 10). The along channel velocity is strongly dependent on water depth such that the deeper the water, the larger its amplitude (Fig. 9). The maximum flood and ebb (flood, ebb) on the East, West, East–West, and Diagonal Transects are (1.3, 1.1), (1.1, 0.9), (1.3, 1.1), and (1.1, 0.9) m/s, respectively. Obviously, the maximum flood is 0.2 m/s stronger than the maximum ebb on all the four transects, indicating a consistent flood dominancy in the inlet. The flood dominant flow in the Sand Shoal Inlet was

![Fig. 9. Surface along channel tidal velocity (m/s) on the four transects. The subplots (a), (b), (c), and (d) are for the East (line CD in Fig. 6), West (line AB in Fig. 6), East–West (line AD in Fig. 6), and Diagonal (line BC in Fig. 6) Transects, respectively. Below each subplot is the water depth along the transect.](https://example.com/fig9.png)
first noticed by Brumbaugh (1996) from 25-h observations at a single position with a hand held current meter. The result here suggests that the flood dominancy appears to be true in at least most of the inlet.

Unlike the along channel surface velocity, the cross channel surface velocity has two modes for all four transects (Fig. 10). In general, the maximum and minimum of the cross channel velocity are located on two sides of the channel, rather than at the deepest water as the along channel velocity. On the East Transect (Fig. 10a), the southern half has a strong positive (roughly northward) flow most of the time and the northern half has a negative (roughly southward) flow most of the time. This indicates a (mean) convergence during most of the tidal cycle likely caused by the constriction of the inlet. Only during 18:00–20:00 UTC, is the convergence replaced by a weak divergence. The convergence during different tidal

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**Fig. 10.** Surface cross channel tidal velocity (m/s). The subplots (a), (b), (c), and (d) correspond to the four transects as in Fig. 9. Unlike the along channel velocity, the cross channel velocity often maximizes itself not in the deepest water but over the major bathymetry changes.
phases in the center of the East Transect is consistent with the observed fronts (Fig. 6).

The surface cross channel flow on the West Transect (Fig. 10b) is quite different from that on the East Transect. The entire transect has a positive (northward) cross channel flow during the first 4 h and then a negative (southward) cross channel flow for about 6 h. And for the last 1–2 h most of the transect has a positive flow again. The East–West Transect (Fig. 10c) has a cross channel velocity distribution similar to some degree to that of the East Transect (Fig. 10a) with the southern \( \frac{2}{3} \) of the transect having a positive (northward) velocity and the rest \( \frac{1}{3} \) having a negative velocity during the first 4–5 h. The flow then reverses for about 6 h and then repeats the earlier pattern for the last 2–3 h. This flow pattern suggests an alternation between a strong convergence and a strong divergence in the center of the channel. The difference between the East–West and the East Transects is that the former does not have the persistent convergence. In addition, the positive and negative velocities across the channel on the East–West Transect is approximately symmetric. The cross channel flow pattern on the diagonal Transect (Fig. 10d) is similar to that on the West Transect: a uniform positive flow for 4–5 h and then a uniform negative flow for 5–6 h and finally a mostly positive flow for the rest of the tidal cycle. The positive flow at the southern end of the transect reached 1.0 m/s for about 3 h, implying a strong convergence. Fronts were also observed at the southern end of the transect (Fig. 6).

5.3. Bottom friction and pressure gradient

The mechanism of tidal convergence fronts discussed earlier (Fig. 2) requires a strong bottom friction. Here we examine the dependence of velocity amplitude and phase on water depth and estimate the bottom drag coefficient to verify the importance of bottom friction. Fig. 11 shows the along channel velocity amplitude and phase as a function of water depth from the West Transect.

![Graph](image-url)

Fig. 11. The along channel surface tidal velocity amplitude (a) and phase (b), as functions of water depth along the West Transect which is roughly perpendicular to the main channel.
The West Transect is chosen here because it is nearly perpendicular to the axial direction of the channel. The phase lag between shallow and deep waters (Fig. 11b) should be compared only on the same cross channel section. Velocity amplitude at 20 and 3 m isobaths are 1.0 and 0.2 m/s, respectively. Velocity phase in shallow water (3 m) leads that in deep water (10 m and beyond) by 12–15° (24–30 min): flood and ebb in shallow water occur earlier than that in deep water along the same cross channel transect. The decrease of velocity amplitude with water depth and the lead of velocity phase in shallow water suggest a strong frictional flow. To further quantify the bottom friction, we use the “phase-matching” algorithm introduced in Li and Valle-Levinson (1999) to estimate the drag coefficient. Basically, the technique applies to narrow tidal channels in which the pressure gradient is along the axial direction. The phase lead of the along channel velocity is projected to the shore (zero depth) based on a Taylor series expansion to estimate the phase of the pressure gradient from the momentum equation. The momentum equation is then used to obtain an optimal estimate of bottom drag coefficient as a function of water depth. Fig. 12a shows the calculated drag coefficient, which varies from 0.004 in 3 m depth to 0.002 in 23 m. The error of the estimate is about ±0.0003. The spatial average value is about 0.0026, typical for unstratified shallow water flows (Proudman, 1953). The doubling of the drag coefficient in shallow water may explain the relative long lead (~ 30 min) in velocity over shallow water, considering the narrowness of the inlet (~1 km). Fig. 12b shows the amplitude of the along channel pressure gradient as a function of water depth. It shows that the pressure gradient is basically constant (~1.6 × 10⁻⁴ m/s²) across the transect, indicating that the assumption used to obtain the drag coefficient is valid and Fig. 12a is correct to the first-order approximation.

5.4. Velocity shears

We then calculate the transverse shear (which roughly has the same magnitude of the relative vorticity) of the along channel velocity, ∂u/∂y,
where \( u \) is the along channel velocity and \( y \) is the cross channel distance measured from the southern end of each transect. A velocity shear is required for the density driven axial convergence front (Sarabun, 1980; Nunes and Simpson, 1985). Here in a barotropic flow environment the strength of the shear will not only demonstrate the strength of bottom friction (vorticity generating force) but also provide a comparison with the planetary vorticity (\( \sim 10^{-4} \text{ s}^{-1} \)) to determine the importance of the planetary vortex tilt effect. The surface velocity shear has 2–3 modes (Fig. 13). During ebb the southern (northern) shoal has a positive (negative) shear. During flood, the shear at a given location changes sign. There is an exception at the end of the Diagonal Transect which shows a negative (positive) shear during (ebb) flood at the southern end (Fig. 13d). This is apparently caused by a strong along channel flow at the constriction of the inlet. Note that the East Transect is at the narrowest place of the inlet (Fig. 4). Considering the extensive shallow shoals south of the main

![Fig. 13](image-url)
channel (Fig. 6) and the v-shaped constriction east of the East Transect (Fig. 4), the strong shear over the southern shoal at the entrance appears to be reasonable (Fig. 13d).

The maximum of the shear is at least $10^{-3}$ s$^{-1}$ except at the southern end of the Diagonal Transect where the shear is about $10^{-2}$ s$^{-1}$. The result suggests that the relative vorticity is much larger than the planetary vorticity ($10^{-4}$ s$^{-1}$) by 1–2 order of magnitudes. The planetary vorticity tilt is therefore unimportant here in generating the tidal convergence fronts observed. This conclusion is consistent with the fact that the fronts occurred on both sides of the channel at the same time.

5.5. Convergence and divergence

To further quantify the transverse convergence of velocity, we now calculate $\partial v/\partial y$ and its temporal variations on the four transects, where $v$ is the cross channel velocity. The calculation reveals that more than $\frac{2}{3}$ of the East Transect is convergent for most of the time (Fig. 14a) with a maximum convergence of about $3 \times 10^{-3}$ s$^{-1}$ at

Fig. 14. Lateral divergence/convergence (s$^{-1}$) of flow along the transects. Positive (divergence) and negative (convergence) flows are indicated by solid and dashed lines, respectively. The subplots (a), (b), (c), and (d) correspond to the four transects as in Figs. 9 and 10.
near the center of the transect. Consulting Fig. 6, the observed front positions are mostly within the southern \( \frac{2}{3} \) of the transect. The two front positions closest to the end appear to be at the edge of the divergent zone. However, considering the estimated ±40 m average observational error in position, they are still within a reasonable range. Unlike at the southern end, we did not observe any fronts on the northern end of the East Transect. This is consistent with the mostly divergent flow at the northern end. In contrast, on the West Transect (Fig. 14b), most of the convergence is on the northern half of the transect during ebb. The strongest convergence \( (2.5 \times 10^{-3} \text{ s}^{-1}) \) is in the first few hours of the observations. Although there is a weaker convergence \( 10^{-3} \text{ s}^{-1} \) after 18:00 UTC on the shallow southern portion of the transect, there is no recorded front positions there. The convergence on the East–West Transect (Fig. 14c) appears to be weaker \( (\sim 1.6 \times 10^{-3} \text{ s}^{-1}) \) than both the East and West Transects. The convergence is mostly centered at the mid-channel. Only 2 recorded front positions (Fig. 6) were close to the transect and these positions appear to be consistent with the convergence (Fig. 14c). The Diagonal Transect experiences mostly convergence with a maximum convergence of \( 10^{-2} \text{ s}^{-1} \) at the southern end (Fig. 14d). This is the strongest convergence in the whole area. Several front positions are recorded at the very end of the transect. Most other recorded front positions near this transect are in the main channel (Fig. 6) which is mostly convergent.

It should be noted that in the calculations, we have ignored the along channel variation of the cross channel velocity on the East–West and Diagonal Transects. These two transects are not perpendicular to the main channel and the calculated convergence can only provide a crude estimate and a relative reference. The result could be accurate if there were no along channel variation of the cross channel velocity. Therefore, the maximum convergence on the Diagonal Transect may be over estimated. The calculated convergence on the East and West Transects, however, are most reliable because they are roughly perpendicular to the main channel. The calculated results for the East–West, and Diagonal Transects are still included here to provide relative information on each of these transects to identify zones of convergence and divergence.

In Fig. 15, we show the surface tidal ellipses on the four transects. The “up” and “down” directions in the figure correspond to the upstream and downstream flows along the main channel. Intertidal convergence and divergence can be identified

![Fig. 15. Tidal ellipses along the four transects. The subplots (a), (b), (c), and (d) are for the East, West, East–West, and Diagonal Transects, respectively. The solid and dashed lines indicates clockwise and counterclockwise rotations, respectively.](image)
qualitatively by this figure. The tilts of the major axes of the tidal ellipses are quite obvious on the East Transect (Fig. 15a) and slightly weaker on the East–West Transect (Fig. 15c). The tilt of the axes on the West (Fig. 15b) and Diagonal (Fig. 15d) Transects may be the weakest. Considering the magnitude of the tidal flow (maximum $\sim 2 \text{ m/s}$), the tilts of axes have resulted in significant net convergence or divergence velocities ($\sim 20 \text{ cm/s}$ or more) on the East Transect. In comparison, the phase difference ($\sim 12–15^\circ$ or 24–30 min) in tidal velocity can result in a convergent velocity amplitude of up to $\frac{1}{6}$ of the cross channel velocity or about 10–15 cm/s.

To better visualize the convergence for both intertidal motion and the mean flow field, we have also applied the harmonic-statistic analysis to the rectangular grid system (Fig. 7a) and obtained the tidal ellipses and mean flow distributions for both surface velocity and depth averaged velocity (Fig. 16). Both the intertidal and mean convergences within the channel are very strong.
The depth averaged intertidal flow (Fig. 16c) has a similar pattern as that on the surface (Fig. 16a) whereas the surface mean convergence (Fig. 16b) is significantly stronger than that of the depth-averaged flow field (Fig. 16d) on the East Transect. The surface mean convergence on the East Transect reaches more than 50 cm/s where fronts are also visually the strongest.

5.6. Quality of analysis

The statistics of the harmonic-statistic analysis on the four transects are summarized in Table 1. The standard deviation for the along channel velocity for the East, West, East–West, and Diagonal Transects are about 15, 9, 13, and 13 cm/s, respectively. Although the cross channel velocity is smaller, its standard deviation is about the same as the along channel velocity: 13, 10, 14, 13 cm/s for the four transects, respectively.

The $R^2$ value for the along channel velocity is very close to 100%, which indicates a strong tidal signal in the along channel velocity. In contrast, the tidal signal in the cross channel velocity is weaker. The $R^2$ value for the cross channel velocity for the four transects are about 53%, 83%, 60%, and 76%, respectively. The particularly low value (53%) for the East Transect is probably caused by the strong constriction, indicating an irregular (nontidal) variation in the region. In a sense, the constriction at the East Transect magnifies any small variation propagated from inside the inlet during ebb or from outside of the inlet during flood because the confined area may concentrate the energy much like the amplification of tidal amplitude when the tidal wave is propagated from the deep ocean to the shallow water.

### Table 1

<table>
<thead>
<tr>
<th>Transect name</th>
<th>East</th>
<th>West</th>
<th>East–West</th>
<th>Diagonal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_u$ (m/s)</td>
<td>0.147</td>
<td>0.090</td>
<td>0.134</td>
<td>0.134</td>
</tr>
<tr>
<td>$\sigma_v$ (m/s)</td>
<td>0.130</td>
<td>0.100</td>
<td>0.137</td>
<td>0.133</td>
</tr>
<tr>
<td>$R^2_u$ (%)</td>
<td>93.14</td>
<td>96.05</td>
<td>93.36</td>
<td>94.41</td>
</tr>
<tr>
<td>$R^2_v$ (%)</td>
<td>52.76</td>
<td>82.71</td>
<td>60.16</td>
<td>75.91</td>
</tr>
</tbody>
</table>

6. Concluding remarks

From the above analysis we may conclude that the fronts observed in Sand Shoal Inlet is caused by frictional tidal motion strongly affected by bathymetry and geometric constriction at the entrance. The planetary vorticity is at least one order of magnitude smaller than the relative vorticity which suggests that the Coriolis force is unimportant and the tilt of the planetary vortex may be neglected. In contrast, the intertidal convergence caused by the phase lead of the velocity in shallow water results in a net convergent velocity of 10–15 cm/s. The intertidal convergence caused by the tilt of the major axes of the tidal ellipses results in a net convergent velocity of at least 20 cm/s. The maximum subtidal convergence velocity on the East Transect is found to be more than 50 cm/s. Although these intertidal and subtidal convergences usually do not maximize at the same location at the same time, together they combine to generate strong fronts in the axial direction that lasted for hours in a number of locations. Perhaps the most remarkable feature of this type of axial fronts, which distinguishes itself from the most documented estuarine fronts, is that the strong convergence is formed without the existence of a noticeable density gradient. In estuaries and coastal embayment where both tides and density gradients are important and where cross-channel bathymetry change is present, the frontal genesis can be a combination of several processes including those discussed in this paper.

In addition to the discussion of frontal generating mechanisms, the current study may also provide some useful techniques and algorithms for future studies. First, we have demonstrated that vessel-based ADCP observations can be used to find not only the harmonic constituents of tidal velocity but also those of tidal elevation at the same time. Li et al. (2000) used a vessel-towed ADCP to obtain the distribution of harmonic constituents of elevation along a 16-km transect at the lower Chesapeake Bay. Whereas this work has extended the method to include the constituents of both elevation and velocity. This makes it possible to (1) correct the vertical coordinate and obtain
correct velocity profiles and transport in true earth-coordinate, and (2) adequately describe the tidal characteristics (both elevation and velocity) with just a single vessel-based ADCP. In comparison, an ADCP with a pressure sensor deployed at the bottom can also yield tidal constituents of both elevation and velocity but only at one location. A vessel-based ADCP can provide spatial variations of the harmonic constituents of both elevation and velocity. Previous studies using vessel-based ADCPs have ignored the surface elevation and thus could not provide velocity profiles with correct vertical coordinates and provide a complete description of tidal characteristics (e.g. whether the tide is a progressive or standing wave) without information from a nearby tide gauge.

Second, the harmonic-statistic analysis presented here for both elevation and velocity profiles have been proven to be a useful tool to assess the statistical characteristics of data and the quality of the observations. The analysis can be applied in different grid systems: along a transect or over a rectangular area. The spatial variations of both the harmonic constants of tides and their statistics can be calculated, making it possible to determine the places of different variabilities either due to the nature of the problem or the observation errors.

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References


Chunyan Li is an Assistant Professor of Physical Oceanography at the Skidaway Institute of Oceanography, USA. He holds a B.S. in Atmospheric Physics from the University of Science and Technology of China, a M.S. in Physical Oceanography from Institute of Oceanology of China and a Ph.D. in Oceanography from the University of Connecticut. His research interests include observational, theoretical and modeling studies of ocean dynamics. Specifically, he is interested in ocean tides and coastal water dynamics, interaction of barotropic and baroclinic motions, coastal ocean and estuarine circulation, including transport and mixing processes, objective or inverse analyses, parameter estimations and the inversion of flow field from limited observations in coastal–estuarine waters, innovation of observational techniques and data analysis techniques, satellite remote sensing in coastal oceanography, MODIS Sea Surface Temperature (SST) and its seasonal variability and climatology of coastal seawater, and wind- and tide-driven cross-shelf transport processes over the South Atlantic Bight area (extending from the inner shelf to the outer-shelf.)