Influence of Lateral Advection on Residual Currents in Microtidal Estuaries

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ABSTRACT

The influence of nonlinear lateral advection on estuarine exchange flow is examined with a scaling analysis and eight groups of idealized numerical experiments. Nonlinear lateral advection is related to the linkage between lateral circulation and the lateral shear of the along-estuary flow. The relative contribution of lateral advection to the overall dynamics of a microtidal estuary is found to be a function of width and depth, and of vertical mixing. Lateral advection is dynamically important in narrow and deep estuaries, particularly under relatively weak vertical mixing. The relative importance of lateral advection and the earth’s rotation on estuarine dynamics can be evaluated in terms of the nondimensional Rossby and Ekman numbers ($R_o$ and $E_k$). Lateral advection is most effective at large $R_o$ and small $E_k$ and is negligible at small $R_o$ and large $E_k$. As expected, the earth’s rotation is most significant at small $R_o$ and $E_k$, and is negligible at large $R_o$ and $E_k$. Under the influence of lateral advection and the earth’s rotation, the lateral structure of estuarine exchange flows is a function of $R_o$ and $E_k$. In some instances, the exchange flow is vertically sheared and in others it is laterally sheared. Classical estuarine dynamics, which yields vertically sheared exchange flows, occurs at intermediate $R_o$ and $E_k$. The main role of lateral advection is to reduce lateral variability of estuarine exchange flow and generate a vertically sheared, two-layer exchange flow structure.

1. Introduction

For several decades it has been assumed that the basic, lowest-order, hydrodynamics of the along-estuary flow is determined by the balance between pressure gradient and stress divergence (or frictional effects; e.g., Pritchard 1956). This linear momentum balance forms the basis for analytical solutions of estuarine circulation (Hansen and Rattray 1965; Chatwin 1976; Officer 1976; MacCready 2004). Although this approach neglects tidal oscillations, lateral structure, and assumes constant vertical mixing, the simplified dynamics can describe the essential structure of estuarine exchange flow. Over a flat bottom, the flow pattern resulting from this linear momentum balance consists of a vertically sheared flow with near-surface outflow and inflow underneath (i.e., two-layer structure).

Bathymetric variations across the estuary can alter the conventional two-layer exchange flow. Over a triangular section, inflow appears in the deepest part and extends to the surface, whereas outflow develops over shoals (Wong 1994). In other words, the vertically sheared exchange flow that appears over a flat bottom becomes horizontally sheared over laterally varying bathymetry. The inclusion of earth’s rotation in the basic estuarine dynamics explains different exchange patterns as a function of the Ekman number, $E_k [E_k = A_z/(fH^2)$, where $A_z$ is the vertical eddy viscosity, $H$ is water depth, and $f$ is the Coriolis parameter], which illustrates the competition between friction and the earth’s rotation (Kasai et al. 2000). With large frictional influences ($E_k > 1$), the current pattern agrees with Wong’s results. As frictional effects decrease ($E_k < 1$), the exchange flow pattern tends to become two layered but may be asymmetrically distributed across the section. The relative importance of the earth’s rotation in Kasai et al.’s analysis is determined by the basin’s depth, whereas it is traditionally assessed from the basin’s width (Pritchard 1952). The basin width can be expressed nondimensionally as the Kelvin number, $K_e$, which is the ratio of basin’s width to the internal radius of deformation (e.g., Garvine 1995). The pattern of estuarine exchange flow can also be characterized as a function of the Ekman and Kelvin numbers (Valle-Levinson 2008). Under high-friction conditions ($E_k > 1$), the flow pattern is independent of $K_e$.

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and is horizontally sheared, in agreement with Wong’s results. Under weak friction conditions \((E_k \ll 1)\), the flow pattern becomes vertically sheared for low \(K_v\) and horizontally sheared for high \(K_v\). For intermediate \(E_k\), the flow pattern is both horizontally and vertically sheared for all \(K_v\).

The above analytical theories depicting estuarine exchange flow patterns result from linear hydrodynamics. However, it has been recognized that the nonlinear advective terms, related to lateral circulation in estuaries, may be as important as the pressure gradient and are crucial to the basic hydrodynamics (Lerczak and Geyer 2004). The lateral structure of tidally averaged lateral advection \((wu_y + wu_z\), where \(u, v, w\) and \(w\) are velocity components in along-estuary, cross-estuary, and vertical directions, respectively, and the subindex indicates partial differentiation) could be of similar shape to the pressure gradient and could help reinforce the two-layer estuarine circulation. In an estuary with asymmetric transverse variability of bathymetry, the tidally averaged lateral advection terms act as a driving force for reducing the lateral structure of the estuarine exchange flow. Namely, lateral advection enhances outflows over shoals and reduces the inflow in the deep channel (Cheng et al. 2009). Nonetheless, the role of lateral advection terms on estuarine exchange flow remains to be explored. Still missing are criteria to determine the relative importance of lateral advection in the along-channel hydrodynamics and how lateral advection modifies the pattern and strength of estuarine exchange flow predicted by linear dynamics.

To address these issues, this study presents a series of numerical experiments using an idealized estuary. The numerical setup was inspired by the idealized numerical study of Hetland and Geyer (2004), which examined the longitudinal estuarine dynamics without lateral variability. In this study, a triangular section of an estuary channel is analyzed to include the effects of transverse bathymetric variations. The most notable simplification of this modeling approach, relative to real estuaries, is the exclusion of tides that can significantly modify classical theories of gravitational estuarine circulation. Since this study is an extension of classical estuarine dynamics, it is limited to microtidal estuaries and the influences of tidal mixing must await future studies. Although this idealized study may be limiting in terms of its quantitative applications, it allows an assessment of the lateral advection role on residual estuarine circulation and it helps to understand nonlinear processes in estuarine hydrodynamics.

The remainder of this paper is structured as follows: in section 3, the setup of the numerical model is described. In section 4, numerical experiments are undertaken to 1) examine the effects of width, depth, and friction (as revealed from dimensionless parameters) on the relative contribution of lateral advection to along-estuary dynamics; 2) explore criteria to determine the relative importance of lateral advection and the earth’s rotation in estuarine dynamics; and 3) investigate the influence of lateral advection on the exchange flow pattern. In section 5, the discussion concentrates on 1) comparing analytical theories of estuarine exchange flow to these numerical results; 2) determining the influence of lateral advection on the strength of exchange flow; 3) underscoring the importance of transverse bathymetric variability; and 4) an initial examination of the influence of tidal forcing. Section 6 summarizes the main findings of the study.

2. Scaling the relative importance of lateral advection

Owing to the difficulty in solving nonlinear momentum equations, the relative importance of lateral advection on the along-estuary dynamics is assessed first by scaling lateral advection against friction and rotation terms in the momentum balance. Neglecting along-channel variability of the estuarine exchange flow, the along-estuary momentum equation at steady state can be written as

\[
v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - f v = -g \frac{\partial \eta}{\partial x} + \frac{g}{\rho_0} \frac{\partial \rho}{\partial x} z + A \frac{\partial^2 u}{\partial z^2},
\]

and the continuity equation as

\[
\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,
\]

where \(\eta\) is the water level, \(\rho\) is the water density, and \(\rho_0\) is a reference density. According to (2), the scale \(W\) of the vertical velocity can be expressed as \(W = VH/B\). Thus, both \(\nabla u \delta y\) and \(\nabla u \delta z\) can be scaled as \(VU/B\), where \(U\) and \(V\) are magnitudes of along-channel and cross-channel velocities, respectively; \(H\) is the maximum water depth of the cross-channel section; and \(B\) is the width of the channel. The scales of lateral and vertical advection terms indicate that the two terms are of the same order of magnitude. Previous numerical studies have shown that lateral advection and vertical advection are correlated to each other and need to be combined together for assessing their contribution to the longitudinal momentum (Lerczak and Geyer 2004). Therefore, the lateral advection used in this study takes into account the sum of lateral and vertical advection terms.
The contribution of lateral advection relative to Coriolis forcing may be explored through the Rossby number \( R_o \):

\[
R_o = \frac{U}{fB}.
\]  

(3)

When Coriolis acceleration is negligible, the relative importance of lateral advection may be assessed by the competition between lateral advection and the frictional term, which can be expressed as an estuarine Reynolds number \( R_e \) that includes the geometry of the cross-channel section:

\[
R_e = \frac{UH^2}{A_z B}.
\]  

(4)

The magnitude of \( U \) in the estimate of (4) can be determined using the analytical solutions of Chatwin (1976) and Officer (1976):

\[
U = \frac{gH^3 \Delta \rho}{48 \rho_0 A_z L}.
\]  

(5)

where \( L \) is the estuary’s length. Freshwater velocity is assumed to be small compared to the estuarine circulation velocity and is neglected. Substituting Eq. (5) into Eqs. (3) and (4), both \( R_o \) and \( R_e \) are determined by \( H, B, \) and \( A_z \) (i.e., \( H/BA_z \)) for a given Coriolis parameter and along-channel density gradient. Large values of \( R_o \) and \( R_e \) represent pronounced contributions of lateral advection. From (3) and (4), it can be gleaned that lateral advection is important for narrow (small \( B \)) and deep (large \( H \)) estuaries (large aspect ratio \( H/B \)), particularly under weak mixing (small \( A_z \)). Combining Eqs. (3) and (4), a relationship among \( E_k, R_o, \) and \( R_e \) is obtained:

\[
R_e = \frac{R_o}{E_k}.
\]  

(6)

This relationship shows that the relative importance of lateral advection can be represented in terms of \( R_o \) and \( E_k \). These two nondimensional numbers have been used also to describe the relative importance of the earth’s rotation. Accordingly, the role of lateral advection and the earth’s rotation on estuarine hydrodynamics may be evaluated in the framework of \( R_e \) and \( E_k \). This premise is explored next with numerical experiments.

### 3. Numerical model description

The Regional Ocean Modeling System (ROMS) was used in this idealized modeling study. The model is a free-surface, hydrostatic, primitive-equations ocean model that uses stretched, terrain-following vertical coordinates and orthogonal curvilinear horizontal coordinates on an Arakawa C-grid (Haidvogel et al. 2000). The model domain is designed as an estuary-shelf system (Fig. 1) following the studies of Hetland and Geyer (2004) and Chen and Sanford (2008). The portion of the domain corresponding to the estuary is straight, 300 km long, and has no along-channel bottom slope. The cross-channel section has a triangular shape with a minimum depth of 1 m on the banks. The width and maximum depth of the channel are adjustable. A freshwater discharge with fixed section-averaged velocity of 0.02 m s\(^{-1}\) is specified at the head of the estuary. The inflowing river water is prescribed to have zero salinity and a temperature of 15°C, identical to the background temperature set throughout the entire domain. The shelf is 80 km wide and has a fixed cross-shelf slope of 0.05%. The salinity of the coastal ocean is 35 and a southward weak flow (0.03 m s\(^{-1}\)) is specified on the shelf to suppress the bulge of freshwater at the estuary mouth. The coastal ocean is included in the domain to avoid specifying boundary conditions at the estuary mouth, which are usually unknown. Vertical diffusivity and viscosity are set to the same constant value, \( A_z \), throughout the domain. Even though this is restrictive, the purpose of using constant \( A_z \) is to compare these results to analytical solutions of previous studies. This allows assessment of the effects of lateral advection on estuarine hydrodynamics.

The model grid is 200 (along-channel, \( x \) direction) by 80 (cross-channel, \( y \) direction) by 20 (vertical, \( z \) direction) cells. The river has 150 grid cells along the channel and 31 grid cells across the channel. The along-channel grid size (\( \Delta x \)) increases exponentially from the estuary’s mouth (~50 m) to its head (~11 km), providing a highly resolved region near the estuary’s mouth. The cross-channel grid in the estuary is uniformly distributed but its size (\( \Delta y \)) is adjustable such that the width of the
estuary can be controlled. The vertical layers are uniformly discretized. The model runs, from rest, for 70–80 days until reaching steady state. This is achieved when the local acceleration of the along-channel flow \( (\ddot{u}) \) is at least three orders of magnitude smaller than the other terms in the momentum balance. Eight groups of numerical experiments are carried out for a number of different values of width \( B \), maximum depth of the estuary channel \( H \), and vertical eddy viscosity \( A_z \) (Table 1). In all runs \( f \) is 0.0001 s\(^{-1}\). Two more groups include tides.

### 4. Results

Numerical results are analyzed in terms of the relative contribution of each dynamic term to the overall momentum balance in the along-estuary direction and of the lateral structure of the exchange flows. For each numerical experiment, the cross section used for analysis is selected in the middle of the estuary. This location maintained a constant position relative to the topography. The momentum terms extracted from each numerical experiment are the total pressure gradient \( P_x (\frac{\partial P}{\partial x}) \), the vertical component of the stress divergence or friction \( (A_z \ddot{u}_z) \), Coriolis acceleration \( (f\nu) \), lateral advection \( (nu_x + wu_z) \), and axial advection \( (uu_x) \). The section average of each momentum term is calculated using

\[
\langle |\phi| \rangle = \frac{1}{A} \int |\phi| \, dA,
\]

where \( |\phi| \) is the absolute value of each momentum term \( \phi \), and \( A \) is the cross-sectional area of the channel. The percentage of each section-averaged term is determined to represent its relative contribution to the overall momentum balance.

#### a. Effects of width, depth, and vertical mixing

Inspired by Eq. (4), three groups of numerical experiments were carried out to evaluate the effects of the width of the estuary \( B \), depth of the estuary \( H \), and vertical mixing \( A_z \) on determining the contribution of lateral advection to along-channel dynamics. The first group explored the effects of \( B \) and consisted of nine experiments with fixed maximum depth \((H = 10 \text{ m})\) and \( A_z (1 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}) \) (Fig. 2a). Results showed that as \( B \) increased, the percentage of lateral advection term decreased from 16.5% for \( B = 0.78 \text{ km} \) to 3.9% for \( B = 30.1 \text{ km} \). A main finding from these experiments is that lateral advection is most influential in narrow estuaries. The Coriolis acceleration term showed the opposite trend to the lateral advection term. The percentage of the Coriolis term increased from 1.9% to 17.6% as \( B \) became wider, which indicated that the earth’s rotation is most important, as expected, for wide estuaries.

The second set consisted of eight experiments with fixed width \((B = 6.2 \text{ km})\) and vertical mixing \((A_z = 1 \times 10^{-3} \text{ m}^2 \text{ s}^{-1})\) and examined the effects of \( H \) (Fig. 2b). The percentage of lateral advection terms increased from 5.8% to 20.3% as the maximum depth of the estuary increased from 5 to 40 m. The percentage of Coriolis term remained approximately constant as the depth increased. This indicated that the earth’s rotation is less sensitive to the estuary depth under fixed estuary width and vertical mixing.

The third group consisted of 10 experiments with fixed width \((B = 6.2 \text{ km})\) and maximum depth \((H = 10 \text{ m})\) and evaluated the variations in vertical mixing (Fig. 2c). The value of \( B \) was chosen on the basis of the first group of experiments such that both lateral advection and Coriolis acceleration could contribute to the along-estuary dynamics. Results showed that the percentages of lateral advection and Coriolis terms decreased as \( A_z \) increased. This indicated that both lateral advection and the earth’s rotation are more important under weak mixing than under strong mixing.

For every experiment of the three groups described above, friction and pressure gradient contributions were dominant, consistent with classical estuarine dynamics.

### Table 1. Setup of numerical experiments. Note: the turbulence closure used is \( k-\omega \).

<table>
<thead>
<tr>
<th>Group No.</th>
<th>( A_z (10^{-4} \text{ m}^2 \text{ s}^{-2}) )</th>
<th>( B (\text{km}) )</th>
<th>( H (\text{m}) )</th>
<th>Cross-channel section</th>
<th>Adective terms</th>
<th>Tides</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>0.78–30.1</td>
<td>10</td>
<td>Triangular</td>
<td>On</td>
<td>None</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>6.2</td>
<td>5–40</td>
<td>Triangular</td>
<td>On</td>
<td>None</td>
</tr>
<tr>
<td>3</td>
<td>0.5–100</td>
<td>6.2</td>
<td>10</td>
<td>Triangular</td>
<td>On</td>
<td>None</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.78–12.4</td>
<td>10</td>
<td>Triangular</td>
<td>On</td>
<td>None</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>0.78–30.1</td>
<td>10</td>
<td>Triangular</td>
<td>On</td>
<td>None</td>
</tr>
<tr>
<td>6</td>
<td>0.5–100</td>
<td>0.78</td>
<td>10</td>
<td>Triangular</td>
<td>Off</td>
<td>None</td>
</tr>
<tr>
<td>7</td>
<td>0.5–100</td>
<td>0.78</td>
<td>10</td>
<td>Triangular</td>
<td>On</td>
<td>None</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>0.78–30.1</td>
<td>10</td>
<td>Triangular</td>
<td>On</td>
<td>None</td>
</tr>
<tr>
<td>9</td>
<td>10–100</td>
<td>0.78</td>
<td>10</td>
<td>Triangular</td>
<td>None</td>
<td>On</td>
</tr>
<tr>
<td>10</td>
<td>Closure</td>
<td>0.78–6.2</td>
<td>10</td>
<td>Triangular</td>
<td>On</td>
<td>On</td>
</tr>
</tbody>
</table>
The importance of axial advection $uw_x$ was minimal, regardless of $B$, $H$, or $A_z$ because of the along-channel uniformity in the estuary. The numerical experiments confirmed the idea that the contribution of lateral advection to estuarine dynamics is a function of the width and depth of the estuary, and of vertical mixing.

b. Relative importance of lateral advection and the earth’s rotation in terms of Rossby and Ekman numbers

The scaling analysis has shown that the role of lateral advection in estuarine dynamics depends on $Re$, $Ro$, and $Ek$ [Eq. (6)]. Different values of $Ek$ were obtained by repeating the first group of experiments using $A_z$ values of $1 \times 10^{-4}$ m$^2$ s$^{-1}$ (this was the fourth group of experiments) and then $A_z$ of $1 \times 10^{-2}$ m$^2$ s$^{-1}$ (this was the fifth group of experiments; see Table 1). The corresponding $Ek$ values for these three groups of experiments were 0.1, 0.01, and 1.0. For the group of experiments with $Ek = 0.01$, the numerical model became unstable when the width of the estuary exceeded 12.4 km, such that only six experiments of this group were executed.

The relative contribution of lateral advection as a function of $Re$ from the three groups of experiments is described in Fig. 3a. The results separated into three groups that corresponded to the different Ekman numbers. For a fixed $Ek$, the percentage of lateral advection term increased as $Re$ increased. This indicated that the relative importance of lateral advection is proportional to $Re$. However, the fact that the lines are nearly parallel
illustrated that a certain percentage of lateral advection contribution may correspond to multiple Reynolds numbers. This means that both $R_o$ and $E_k$ are required to determine the relative importance of lateral advection in estuarine dynamics. The Rossby number dependence (Fig. 3b) showed similar features as $R_o$ for predicting the relative importance of lateral advection. For a fixed $E_k$, the percentage of lateral advection term increased as $R_o$ became large. For a fixed $R_o$, the percentage of lateral advection term increased as $E_k$ decreased, in agreement with Eq. (6). The experiments also confirmed that the relative importance of lateral advection can be determined as a function of $R_o$ and $E_k$.

The effects of the earth’s rotation on estuarine dynamics have been recognized to be determined by Rossby and Ekman numbers as shown by the dashed lines in Fig. 3b. For a fixed $R_o$, the percentage of Coriolis acceleration decreases as $E_k$ increases. Similarly, for a fixed $E_k$ the percentage of the Coriolis term decreases as $R_o$ increases. Because the lateral advection defined in this study includes both lateral and vertical advective terms, the critical value of $R_o$, that is expected to separate the influence of advection relative to Coriolis terms is 0.5 instead of 1.0. The critical values of $R_o$ revealed from the numerical experiments (Fig. 3b) range from 0.2 to 0.6, agreeing with the expected value.

On the basis of the above analysis, the roles of lateral advection and the earth’s rotation on estuarine exchange flow can be depicted using a two-parameter ($R_o$-$E_k$) framework (Fig. 4). Contours have been drawn to represent the relative contribution (in percentage) of the Coriolis term (broken lines) and lateral advection (continuous lines). The contours are obtained using the numerical results from the five groups of experiments depicted thus far. Because of the limited number of numerical experiments, Fig. 4 shows relatively narrow ranges of $R_o$ and $E_k$. The purpose of this figure is to illustrate the conditions under which lateral advection and Coriolis acceleration are important. Accurate limits of each category for wide ranges of $R_o$ and $E_k$ must await further study. Generally, lateral advection is important for large $R_o$, while the earth’s rotation is important for small $R_o$. Around the critical value of $R_o = 0.5$, the two mechanisms make similar contributions (i.e., between 8% and 11% by lateral advection and ~10% by Coriolis for the $E_k$ range explored). The importance of lateral advection and rotation also depends on friction. As $E_k$ increases, the contributions of lateral advection and Earth’s rotation decrease. Lateral advection is most effective at large $R_o$ and small $E_k$, and its importance decreases toward the top left corner of the parameter space, where $R_o$ is small and $E_k$ is large. The importance of Earth’s rotation is greatest at small $R_o$ and $E_k$, and is negligible at large $R_o$ and $E_k$. Particularly, the classical estuarine dynamics (Pritchard’s type) occurs at intermediate $R_o$ and large $E_k$ where both advection and the earth’s rotation can be neglected.

c. Influences of lateral advection on the pattern of estuarine exchange flows

Results of numerical experiments showed that transverse variability in bathymetry affects the lateral structure of estuarine exchange flows because of the relative contribution of lateral advection and Coriolis effects (Fig. 5). Nine numerical experiments were selected from the three groups of experiments (groups 1, 4, and 5) to represent prominent exchange flow patterns in the $R_o$-$E_k$ parameter space (Fig. 5). The along-channel flows were normalized by the maximum inflow. Positive contour values represent outflow and negative values denote inflow. The depth (ordinates in Fig. 5) was normalized by the maximum depth and the cross-channel distance (abscissas in Fig. 5) was normalized by the width of the channel. Note that the numerical experiments portrayed did not include extreme cases with very large or small $R_o$ and $E_k$. Figure 5 displays general trends of flow patterns varying with $R_o$ and $E_k$ rather than depicting all possible lateral structures of estuarine exchange flow.

The exchange flow pattern shows sensitive dependence on $R_o$ and $E_k$. At large $R_o$ (Figs. 5c,f,i), when lateral
advection contributes to the dynamics, the exchange flow is vertically sheared. But as $E_k$ increases, the exchange flow also displays horizontal shears. Under strong influence of Coriolis accelerations (small $R_o$; Figs. 5a,d,g), the exchange flow pattern is horizontally sheared and the outflow on the left (looking into the estuary) is stronger than the outflow on the right. Furthermore, the right branch of outflow tends to vanish under small $E_k$ (0.01) and the exchange flow dynamics approaches geostrophy under very weak friction. At intermediate $R_o$ (around 0.5; Figs. 5b,e,h), the exchange flow pattern is horizontally and vertically sheared, showing influences of both the earth’s rotation and lateral advection. As $E_k$ increases, the dominant flow structure changes from vertically sheared to horizontally sheared. Under large friction ($E_k \gg 1$), the contributions from both lateral advection and Earth’s rotation are diminished and the exchange flow becomes horizontally sheared. The flow pattern of Pritchard’s-type estuarine dynamics (large friction but no rotation; Fig. 5b) shows a similar structure to Wong’s (1994) model. It is also noteworthy that the patterns shown in Fig. 5 are similar to those in Valle-Levinson (2008, his Fig. 3) drawn from an analytical linear model. Observations [e.g., in Delaware Bay (Wong 1994), Chesapeake Bay (Valle-Levinson et al. 2003), and in the Baltic Sea (Umlauf et al. 2007)] show some transverse patterns of gravitational circulation as predicted in Fig. 5, supporting the idealized study.

To further examine nonlinear effects, the nine experiments shown in Fig. 5 were repeated but with the exclusion of advection terms. A notable feature of the flow patterns from the linear numerical model is that none of the exchange flows show a conventional two-layer structure (Fig. 6). At small $R_o$, the flow patterns from the linear numerical model (Figs. 6a,d,g) are almost identical to those from the nonlinear numerical model (Figs. 5a,d,g) because the contribution of lateral advection is negligible (the contribution of the lateral advection term is less than 5%; Fig. 2b) for wide estuaries. At large $R_o$ the influence of the earth’s rotation can be neglected so that the flow patterns from the linear model are horizontally sheared at different $E_k$ (Figs. 6c,f,i). This is consistent with Wong’s (1994) theory but shows differences with Kasai et al.’s (2000) model. Comparing the fully nonlinear results to the corresponding linear runs, it appears that lateral advection reduces lateral variability of estuarine exchange flows and favors the two-layer structure. At moderate $R_o$ (~0.5), the flow patterns from the linear runs are horizontally sheared but with an enhanced left branch of outflow (Figs. 6b,e,h) resulting from the earth’s rotation. The enhancement is most prominent under weak mixing (Fig. 6h). The linear flow pattern under moderate $R_o$ and high $E_k$ (Fig. 6b) is similar to the corresponding nonlinear result (Fig. 5b) and resembles Wong’s results obtained from the balance between pressure gradient and friction (Pritchard’s dynamics). This is because lateral advection and Earth’s rotation can be neglected at moderate $R_o$ and high $E_k$.

The momentum balance of the experiment corresponding to Fig. 5f is shown in Fig. 7 to further examine the effects of lateral advection on estuarine exchange flow. The local acceleration is four orders of magnitude less than the other terms and has a similar pattern as the exchange flow. Hence, the flow is considered to be at steady state. Coriolis acceleration and along-channel acceleration are one order of magnitude smaller than

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**Fig. 5.** Lateral structure of estuarine exchange flow in terms of $R_o$ and $E_k$. Darker areas denote regions of inflows. Shown are nine experiments from the nonlinear numerical model.
lateral advection, pressure gradient, and friction terms, and are less important in the momentum balance. The lateral advection term was moved to the same side of the pressure gradient in the momentum equation (right-hand side) such that its effect on the exchange flow pattern can be evaluated following the similar physical meaning as the longitudinal pressure gradient. Friction does not show the opposite pattern to the pressure gradient as predicted from linear dynamics, but balances the combination of pressure gradient and lateral advection. In a sense, both pressure gradient and lateral advection act as driving forces of estuarine exchange flow while friction is the balancing force. Because of the relatively low mixing, the bottom Ekman layer is thin.

Fig. 6. Lateral structure of estuarine exchange flow in terms of $R_o$ and $E_k$. Darker areas denote regions of inflows. Shown are nine experiments from the linear numerical model. Each case corresponds to those from the nonlinear numerical model (Fig. 5). The model setup of each case of the linear numerical model is the same as that of the nonlinear numerical model but the advection terms are excluded.

Fig. 7. Transverse distributions of major momentum terms in along-channel dynamics. Shown are results from the experiments corresponding to Fig. 5f. Local acceleration is contoured in units of $10^{-19} \text{ m s}^{-2}$. Other momentum terms are contoured in units of $10^{-5} \text{ m s}^{-2}$. 
and is confined to near the bottom. According to the vertical distribution of Fig. 7, lateral advection tends to accelerate landward currents (negative value) near the bottom and reinforce seaward currents near the surface (positive value). Thus, in agreement with Lerczak and Geyer (2004), lateral advection acts in a similar way to the pressure gradient to drive the two-layer estuarine circulation.

5. Discussion

a. Linear theories of estuarine exchange flow

Several analytical linear models have been proposed to describe the lateral structure of estuarine exchange flow in estuaries with transverse variability of bathymetry (Wong 1994; Kasai et al. 2000; Valle-Levinson 2008). Those results have shown fair consistency with natural systems and with this numerical study. Wong’s model predicts a horizontally sheared exchange flow but is only valid under strong friction because of the exclusion of Earth’s rotation. Kasai et al.’s model revealed the relationship between exchange flow patterns and \( E_k \). However, the flow patterns from the linear numerical model (Figs. 6c,f,i) are insensitive to \( E_k \) in contrast to Kasai et al.’s model. In particular, the horizontally sheared flow pattern persists at small \( E_k \). A possible reason is that Kasai et al.’s model assumes geostrophic dynamics for small \( E_k \) and the linear numerical model is not in geostrophic balance because the simulations are not frictionless. Valle-Levinson (2008) extended Kasai et al.’s model by designing an exponential distribution of the transverse slope of sea level and predicted exchange flow patterns in an \( E_k-K_e \) parameter space. Because \( K_e \) is an inverse proxy of \( R_o \), the \( R_o-E_k \) diagram presented here is physically consistent with the \( E_k-K_e \) parameter space.

As shown in the previous section, lateral advection tends to produce a two-layer exchange flow structure. This effect is pronounced at low \( E_k \) and is negligible at high \( E_k \). The exchange flow patterns resulting from nonlinear dynamics therefore vary with \( E_k \) in the same way as Kasai’s model. Also, the relative contribution of lateral advection is important in narrow estuaries and insignificant in wide estuaries such that the flow patterns obtained from nonlinear dynamics vary with estuary width (\( R_o \) or \( K_e \)) in the same way as in Valle-Levinson’s (2008) model. Consequently, the relationship between estuarine exchange flow pattern and \( E_k/R_o \) from nonlinear dynamics is consistent with that predicted from the analytical linear models. On the other hand, it should be pointed out that including lateral advection would modify the transition, in the \( R_o-E_k \) parameter space, between vertically and horizontally sheared flow patterns. In other words, two-layer flow patterns may occur in the region of the \( R_o-E_k \) parameter space where linear models predict laterally sheared flow patterns. This effect can be discerned by comparing Figs. 5 and 6. Valle-Levinson (2008) proposed a classification of exchange flow patterns in the \( E_k-K_e \) parameter space based on an analytical linear model. The numerical experiments portrayed here support such a type of classification. The inclusion of lateral advection generates a similar classification but the limits of each category will be modified, particularly for the region for which two-layer, vertically sheared flows develop.

b. Influences of lateral advection on the strength of estuarine exchange flow

Two groups of experiments (groups 6 and 7; Table 1) based on nonlinear and linear numerical models, respectively, are compared to investigate how lateral advection affects the strength of estuarine exchange flow. Lateral advection is effective in narrow estuaries so the estuary width is chosen as 0.7 km. The maximum depth is 10 m and the vertical eddy viscosity varies from 0.5 to \( 100 \times 10^{-4} \text{ m}^2 \text{ s}^{-1} \). Because the estuary adjusts gravitationally from rest, the along-channel density gradient is not fixed for a pair of nonlinear and linear runs. The strength of along-channel flow cannot be compared directly using modeled velocities. Wong (1994) derived the exchange flow velocity in an estuary with a triangular cross section. The magnitude is proportional to \( gH^2 \Delta \rho/60 \mu O A_L \), which is similar to Eq. (5). Given that the width and depth of the estuary are fixed in the two groups of experiments and that the freshwater velocity is small (0.02 m s\(^{-1}\)), the strength of exchange flows for both flat and triangular bottoms is determined by the along-channel density gradient and vertical eddy viscosity. To simplify the estimation, \( U \) was defined using Eq. (5). The along-channel density difference is computed using the section average of density at the mouth of the estuary and at the section where the section-averaged salinity equals 1.0. The distance between the two sections is the estuary length. For both nonlinear and linear models, the exchange flow becomes weak as vertical mixing increases (Fig. 8). Additionally, the nonlinear numerical model produces stronger estuarine circulation (ordinate in Fig. 8) than the corresponding linear numerical runs. This suggests that lateral advection enhances the strength of estuarine circulation. Furthermore, the difference of exchange flow between nonlinear and linear models becomes larger as eddy viscosity decreases because lateral advection is dynamically most prominent under weak friction.

c. Estuaries with flat bottom

Lateral variations in bathymetry have been recognized to be crucial for lateral advection to make significant
contributions to estuarine dynamics. Kasai et al. (2000) examined effects of vertical mixing on the exchange flow pattern over a flat bottom and showed that the flow pattern agrees with the classical estuarine circulation regardless of friction. In this study, we further investigate if estuary width influences the contribution of lateral advection in estuaries with flat bottom. The first group of experiments was repeated using a rectangular cross-channel section with a depth of 10 m (eighth group of numerical experiments). The vertical viscosity is $1 	imes 10^{-3}$ m$^2$ s$^{-1}$ and the width varies from 0.78 to 30.1 km. The relative contributions of the five terms in the along-channel momentum equation are shown in Fig. 9 as a function of estuary width. Pressure gradient and friction terms dominate, which is Pritchard’s type of estuarine dynamics. Because of constant vertical mixing, the friction term keeps nearly invariable as the width varies. The lateral advection term becomes less important (the percentage is less than 4%) over a flat bottom. As expected, the relative importance of the earth’s rotation increases as the estuary becomes wider whereas the pressure gradient decreases to equilibrate the momentum. The exchange flow patterns show the conventional two-layer structure even though the salinity distribution may show lateral structure in wide estuaries (not shown here). Geostrophic flows were not observed in the experiments because of the relative influence of friction. In conclusion, lateral advection is negligible for estuaries with uniform cross-channel bathymetry and therefore along-channel flows follow linear estuarine dynamics.

### d. Influence of transverse variations in estuarine exchange flow on salt transport

The subtidal salt balance in an estuary is maintained by a downstream transport resulting from freshwater outflow and an upstream transport resulting from down-gradient salt flux (Monismith et al. 2002). The baroclinic exchange flow is often considered a main mechanism associated with the longitudinal salt dispersion and is usually assumed as a vertically sheared flow based on the classical estuary dynamics. Fischer (1972) pointed out that in many real estuaries the most important mass transport mechanism is the net (nontidal) transverse circulation. This study has shown that in estuaries with transverse bathymetric variability, the cross-channel pattern of estuarine exchange flow varies from vertically sheared to horizontally sheared in terms of Rossby and Ekman numbers as shown in Fig. 5. Therefore, it is expected that the transverse variability of estuarine exchange flow may play an important role in determining estuarine salt transport. On the basis of the analytical solution of Hansen and Rattray (1965), the along-channel salinity gradient and estuary length ($X_s$) were found to be dependent on the freshwater water discharge ($Q_f$), vertical mixing, water depth, and estuary width (Monismith et al. 2002):

$$\frac{S_0}{X_s} \sim A_z \frac{Q_f^{1/3} H^{-3} B^{1/3}}{C_0}$$

or

$$X_s \sim A_z^{-1} \frac{Q_f^{-1/3} H^3 B^{1/3}}{C_0}.$$

(8)

where $S_0$ is the salinity of the ocean. Transverse patterns of estuarine exchange flow are determined by estuary width and depth and vertical mixing. This study only examines the influences of estuary width on along-channel salinity gradient (or estuary length) using the results of the first group of numerical experiments (Fig. 10a). The salinity gradient varies with the width of the estuary following a power function. The best-fit power is $-0.43$, that is, close to $-\frac{1}{2}$. The estuary length...
scale is inversely proportional to the salinity gradient, which indicates that wider estuaries have a longer salt intrusion length than that predicted by conventional linear estuarine dynamics. Estuarine circulation is essentially driven by the along-channel density gradient, such that the strength of exchange flow may decrease as estuary width increases. The maximum magnitude of velocity of exchange flow shows a power function of estuary width (Fig. 10b), and the best-fit power is 0.25.

According to Fig. 5 (bottom), as estuary width increases the exchange flow pattern changes from vertically sheared to horizontally sheared. This indicates that transverse variations in estuarine exchange flow can enhance salt intrusion and reduce estuarine circulation.

In the cases of this study, the upstream salt dispersion is associated with estuarine exchange flow, and can be represented by a Fickian diffusion coefficient $D_t$ (Monismith et al. 2002). Following Fischer (1972), the salt flux can be separated into two parts transported by the transverse and vertical shear flows, respectively, and the dispersion coefficients can be obtained by parameterizing the two components of salt flux using Fick’s law:

$$D_t + D_v = \frac{1}{dS/dx} (u_S S_I + u_v S_v),$$

where the subscripts $t$ and $v$ stand for variation in the transverse and vertical directions, respectively. The overbar denotes cross-sectional average. The transverse and vertical dispersion coefficients of the first group of numerical experiments are shown in Fig. 10c. The vertical dispersion coefficient $D_v$ is about 10 m$^2$ s$^{-1}$ and slightly decreases with increasing width. The transverse dispersion coefficient $D_t$ is smaller than $D_v$ at narrow estuaries and significantly increases to be one order of magnitude larger than $D_v$ at wide estuary ($B = 30.1$ km). This confirms the conclusion drawn from Fischer (1972) that the transverse variations in estuarine exchange flow and stratification can greatly enhance $D_t$ over that predicted for a flat bottom cross section. The increasing salt dispersion results in a longer salt intrusion that reduces the along-channel salinity gradient and the strength of estuarine circulation. Since most estuaries exhibit transverse variations of bathymetry and structure of along-channel flow, linear analytical theories of estuarine circulation may underestimate salt dispersion, so laterally sheared flow dispersion needs to be included.

e. Influence of tides

The influence of tides on residual estuarine circulation may include three aspects. First, tides create residual currents that modify density-driven circulation. For typical tidal channels with relatively strong mixing, the Eulerian tidally induced residual currents are seaward at all depths and therefore reinforce the two-layer density-driven currents in the surface waters, but oppose them in the bottom waters while the Lagrangian tidally induced residual currents oppose the two-layer flow in both the surface and bottom waters (Ianniello 1977). In most studies, residual estuarine currents are Eulerian. It is expected that the computed/measured tidally averaged water flux would be larger than river discharge because of the influence of tides. Second, in tidally dominated estuaries, tidal straining destabilizes the water column on flood tides and increases stratification on ebb tides (Simpson et al. 1990). As a result, vertical mixing exhibits prominent changes over a tidal cycle. One-dimensional numerical simulations have shown that the subtidal circulation caused by tidally periodic stratification has the same vertical structure as the expected classical estuarine exchange flow (Stacey et al. 2008). Under the influence of lateral advection and the earth’s rotation, the effects of tidally periodic stratification on estuarine exchange flows will become more complicated. Third, tides affect salt transport because of the effects of tidal dispersion. The gravitational circulation is essentially driven...
by a horizontal density gradient that is determined by the salt structure. In this section, the first two aspects of tidal influence are explored by conducting two groups of numerical experiments (Table 1; groups 9 and 10).

Based on the same model configuration, tides with amplitudes of 0.5 m are introduced at the eastern open boundary of the model. The tidal period is 12 h. The ninth group of experiments examines the influence of tidally induced currents. Constant vertical mixing is applied with $A_z$ values of 0.001, 0.005, and 0.01 m$^2$s$^{-1}$. Tidally averaged currents and lateral advection distributions are shown in Fig. 11. Under weak mixing (Fig. 11e), the estuary exchange flow shows a two-layer structure. As mixing increases, inflows tend to dominate the deep part of the channel and outflows concentrate over the shoals (Figs. 11a,c). This trend is the same as that without tides (i.e., Figs. 5c,f,i). However, it is noted that the outflow is stronger than the inflow and the section-averaged residual water flux is larger than the imposed river discharge. This confirms the results of Ianniello (1977) that the Eulerian tidal residual currents reinforce the outflows near the surface and reduce the inflows near the bottom. Because the experiments include weak tides, tidally induced residual flows have inconsequential effects on the gravitational circulation. In estuaries with stronger tidal currents, however, tidally induced residual currents could dominate the residual estuarine circulation and the estuarine exchange flow patterns may not follow the trend indicated by $R_e$ and $E_k$. Therefore, the results of this study are limited to microtidal estuaries. The tidally averaged lateral advection term is negative near the bottom and positive near the surface, showing a two-layer structure (Figs. 11b,d,f). As mixing decreases, the negative layer tends to be confined to the bottom. When compared to the patterns of exchange flow, lateral advection tends to extend the inflows from the deep part of the channel to the shoals, to reduce lateral variability, and to produce a two-layer structure across the entire channel. This is consistent with the results obtained with implicit tides and those of Lerczak and Geyer (2004).

In the tenth group of experiments, the $k$–$\omega$ turbulence closure is applied to provide temporal variations of vertical mixing. Four experiments are carried out to examine the effects of estuary width. Figure 12 shows tidal averages of salinity, along-channel currents, and lateral advection. As the width increases, the isopycnals are tilted upward to the right side of the transverse section, indicating the effects of the earth’s rotation. The residual currents show two-layer structure and inflows extend to the right bank of the transverse section in wide estuaries. This trend is consistent with the results with implicit tides (Figs. 5g,h,i). The tidally averaged lateral advection shows complicated patterns and its effects on residual circulation are not obvious visually. The patterns of the tidally averaged lateral advection term do...
not seem to reinforce the two-layer density-driven circulation in contrast to the patterns predicted from experiments with constant mixing. This suggests that variable mixing generates complex nonlinear processes. The lateral advection term contributes more than 20% to the total momentum and its contribution is similar to that of the along-channel pressure gradient (Table 2). Obviously, friction is mainly balanced by pressure gradient and lateral advection. In a sense, both pressure gradient and lateral advection act as the driving force of residual currents while friction is the balancing force. As expected, the relative contribution of the Coriolis term increases, and that of lateral advection decreases, as the estuary becomes wider. This indicates that the non-dimensional parameter space \( \left( \frac{Ro}{Ek} \right) \) is still valid to evaluate the relative contribution of lateral advection and the earth’s rotation in microtidal estuaries. However, with realistic turbulent mixing, the conclusion obtained by Lerczak and Geyer (2004) and in this study that lateral advection acts as a driving force for producing two-layer estuarine circulation needs further study, particularly in mesotidal and macrotidal estuaries.

6. Summary

In estuaries with transverse bathymetric variability, lateral advection can play an important role in estuarine circulation. The relative contribution of lateral advection in microtidal estuarine hydrodynamics can be assessed in terms of the Rossby number \( R_o \) and an estuarine Reynolds number \( R_e \), which is the ratio between the Rossby and the Ekman numbers. Both \( R_e \) and \( R_o \) are a function of the width and depth of the estuary and of vertical mixing \( \left( \sim \frac{H}{BAz} \right) \). Idealized numerical experiments confirmed that lateral advection is influential in narrow and deep estuaries, particularly under weak mixing. Through \( R_e \), the relative importance of both lateral advection and the earth’s rotation can be evaluated in an \( R_o - E_k \) parameter space. Lateral advection is most effective at large \( R_o \) and small \( E_k \) and is negligible at small \( R_o \) and large \( E_k \). The earth’s rotation is most significant at small \( R_o \) and \( E_k \), and is negligible at large \( R_o \) and \( E_k \). The classical estuarine dynamics (Pritchard’s type) occurs at intermediate \( R_o \) and large \( E_k \).

### Table 2. Percentages of section-averaged momentum terms in along-channel dynamics in total momentum for the tenth group of experiments.

<table>
<thead>
<tr>
<th>( B ) (km)</th>
<th>( u_t )</th>
<th>( fu )</th>
<th>( uu_x )</th>
<th>( vu_x + wu_z )</th>
<th>( P_x )</th>
<th>( A_x u_{zz} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>2.02</td>
<td>1.86</td>
<td>3.33</td>
<td>26.56</td>
<td>26.07</td>
<td>40.17</td>
</tr>
<tr>
<td>1.5</td>
<td>2.40</td>
<td>3.04</td>
<td>3.86</td>
<td>23.69</td>
<td>27.25</td>
<td>39.75</td>
</tr>
<tr>
<td>3.1</td>
<td>3.20</td>
<td>4.50</td>
<td>4.13</td>
<td>22.84</td>
<td>27.29</td>
<td>38.04</td>
</tr>
<tr>
<td>6.2</td>
<td>2.37</td>
<td>5.24</td>
<td>4.23</td>
<td>21.13</td>
<td>28.78</td>
<td>38.25</td>
</tr>
</tbody>
</table>
Under the influences of both lateral advection and the earth’s rotation, the lateral structure of estuarine exchange flow exhibits distinct variations in terms of \( R_o \) and \( E_k \). When lateral advection is important (large \( R_o \)), the flow pattern shows a two-layer structure and becomes horizontally sheared as \( E_k \) increases. Under the strong influence of Coriolis acceleration (small \( R_o \)), the exchange flow pattern is horizontally sheared with asymmetric outflows. At intermediate \( R_o \), the exchange flow pattern is both horizontally and vertically sheared. Furthermore, as \( E_k \) increases, the dominant flow structure changes from vertically sheared to horizontally sheared. Compared to results of corresponding linear numerical experiments, lateral advection appears to reduce lateral variations of estuarine exchange flow and produces a two-layer flow structure.

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