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PIPE NETWORK ANALYSIS

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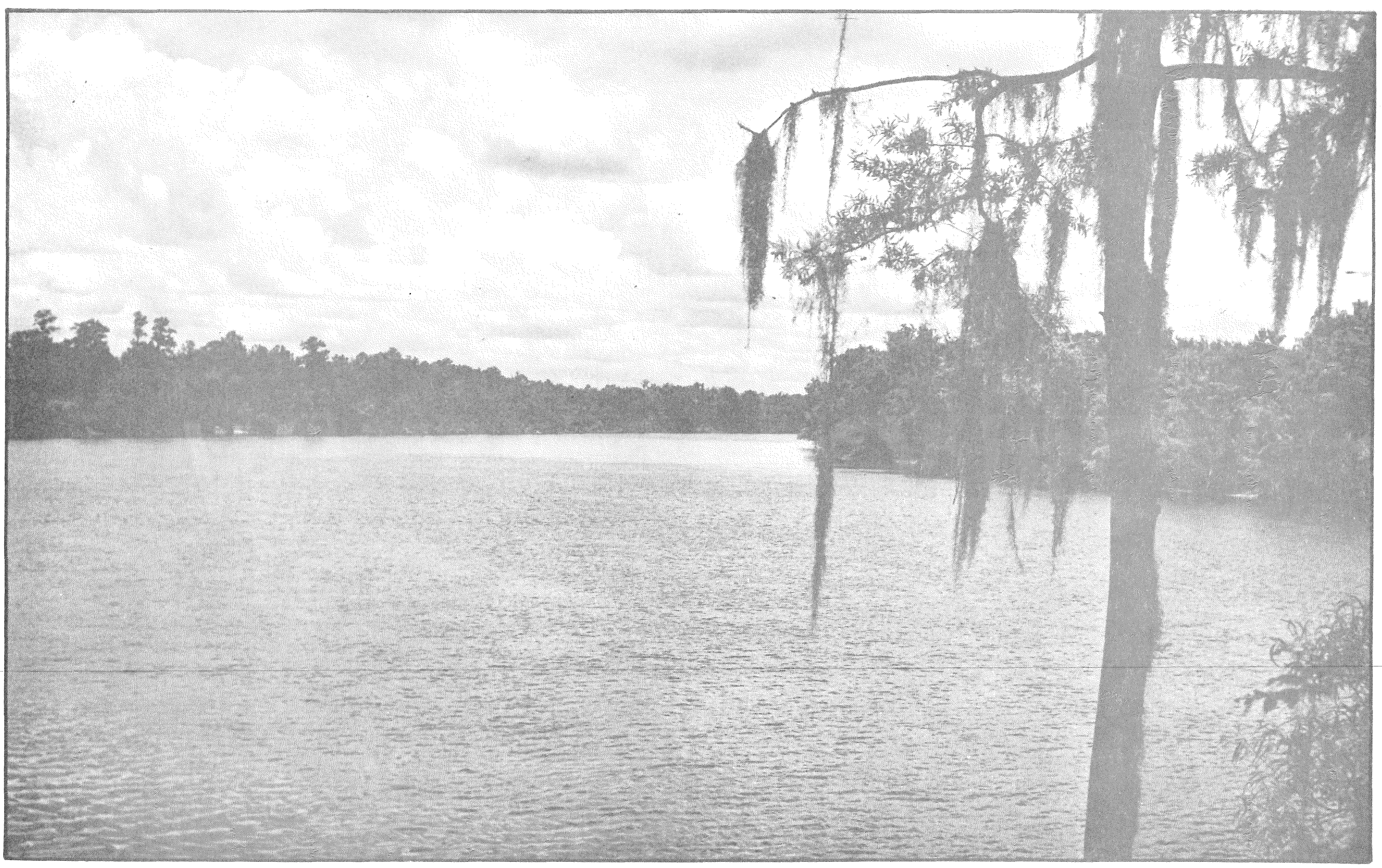

UNIVERSITY OF FLORIDA

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MUN-FONG LEE

NON-THESIS PROJECT REPORT IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE MASTER OF ENGINEERING DEGREE

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ABSTRACT

Analyses of large scale pipe networks are needed whenever significant changes in patterns or magnitudes of demands or supplies occur in municipal water or gas distribution systems. Changes of this nature occur whenever new industrial and residential areas are being developed or new sources of supply are tapped. In the absence of such analytical tools to determine the performance of an existing system under new demands, needlessly large investments are made for larger than necessary pipes, redundant lines or duplicate facilities.

Another cause for concern is the ability of the numerous algorithms to provide reliable results without which deficient engineering judgments may be made in engineering applications dealing with large scale pipe networks. Convergence and reliability problems of most of the algorithms are highlighted after the theoretical background has been presented. As an aid to more effective formulation of the loop and nodal equations, the essential concepts of network theory are also presented together with the fundamental hydraulic principles forming the backbone of the state of the art iterative procedures.

This report concludes with a new approach which employs optimization techniques to solve the pipe network problem as a viable and perhaps more versatile alternative to the widely used iterative methods.

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CHAPTER 1

INTRODUCTION

Analyses and design of pipe networks create a relatively complex problem, particularly if the network consists of a range of pipes as frequently occurs in water distribution systems of large metropolitan areas, or natural gas pipe networks. In the absence of significant fluid acceleration, the behavior of a network can be determined by a sequence of steady state conditions, which form a small but vital component for assessing the adequacy of a network. Such an analysis is needed each time changing patterns of consumption or delivery are significant or add-on features, such as supplying new subdivisions, addition of booster pumps, pressure regulating valves, or storage tanks change the system.

The steady state flows of a network are governed by the laws of conservation of energy and mass and the classical pipe network analysis problem is to establish the steady state flows and pressures in a full flow closed conduit network of known physical characteristics. Due to the complexity and the inherent non-linearity of networks, solving the network analysis problem is not a trivial exercise.

For over four decades, a number of algorithms have been developed since the pioneering work of Hardy Cross. All of these techniques are iterative in nature, differing only in the method in which an estimate of the true solution is obtained. A recent study (Collins, Cooper, Helgason and Kennington, 1978) uncovered a new approach to the pipe network analysis problem using optimization techniques which represent a radical departure from the traditional state of the art methods. This report attempts to provide a comprehensive write-up of the theory behind some of the more commonly used algorithms and their efficiency and reliability.

1.1 PROBLEM DEFINITION

A pipe network is physically a collection of interconnected elements such as pipes, pumps, reservoirs, valves, and similar appurtenances. Mathematically, the network is represented as an edge set consisting of pipes, pumps, valves and similar elements and a node set comprising reservoirs and element intersections. In most of the elements, a unique functional relationship between pressure and flow exists. Pressure, in incompressible flow networks, can be expressed in terms of an equivalent hydraulic head, a terminology which will be adopted throughout this report as is standard practice.

The steady state condition of a network can be completely defined by the head at each node and the flow in each element. Having determined this unique set of flows/ heads for a given set of inputs and withdrawals, all other quantities of interest can be deduced therefrom.

1.2 SIGNIFICANCE

Steady state network analysis is a basic tool in water distribution system management and design. It can also be used to develop operating policies and strategies to not only reduce operating costs but also increase reliability and reduce water wastage (Brock, 1970; Hudson, 1974; Rao et al. 1974, 1977; Shamir, 1974; Bree et al. 1975). Application of steady state network analysis in on-line system control is also receiving growing attention (Brock, 1963; Hudson, 1973; McPherson et al. 1974; Rao et al. 1974; Gerlt and Haddix, 1975; Eggener and Polkowski, 1976).

1.3 MOTIVATION

Since Hardy Cross first provided a solution for the pipe network analysis problem, three general methods which are widely used today, have evolved:

(i) Hardy Cross (Hoag and Weinberg, 1957; Graves and Branscome, 1958; Adams, 1961; Brock, 1963; Bellamy, 1965; Dillingham, 1967; Fietz, 1973; Williams, 1973; Chenoweth and Crawford, 1974; Eggener and Polkowski, 1976)

(ii) Newton-Raphson (Martin and Peters, 1963; Shamir and Howard, 1968; Liu, 1969; Epp and Fowler, 1970; Zarghamee, 1971; Lam and Wolla, 1972; Lemieux, 1972; Donachie, 1973; Rao et al, 1974,1977)

(iii) Linearization (McIlroy, 1949; Marlow et al. 1966; Wood, and Charles, 1972; Fietz, 1973; Collins and Johnson, 1975).

These methods solve a set of non-linear simultaneous equations iteratively beginning with an initial trial solution. The iteration is complete when a new solution differs from the trial solution by less than a specified amount; otherwise, the new solution becomes the trial solution and the procedure is repeated. Differences in the above methods arise because of the strategies used to determine a new solution.

In view of the iterative nature of these methods, large scale networks with hundreds of nodes and elements require considerable computer efforts to solve. The choice of algorithm therefore, depends on the computational speed and reliability of a particular solution procedure.

Matrices associated with water distribution networks, like most man-made systems, are sparse. One of the keys to faster convergence and hence to greater computational efficiency and perhaps reliability for most, if not all, algorithms is the use of sparse matrix techniques in the solution procedures (Tewarson, 1973).

In the following chapters, most of the essential tools required for the analysis of incompressible flow in pipe networks are presented. Chapter 2 introduces graph theory which is useful in the formulation of pipe network simulators and also includes fundamental hydraulic principles governing pipe networks to provide the necessary groundwork for the development of the loop and node system of equations. In Chapters 3 and 4, methods for solving these systems of non-linear equations are described. Alternative mathematical approaches and the writer's computational experience are presented in Chapter 5.

* * * * *

CHAPTER 2

NETWORK ANALYSIS & PIPE NETWORK HYDRAULICS

There has been growing awareness that certain concepts and tools of network theory are very useful in the analysis of pipe networks especially in the formulation of computer simulators. The theory of network analysis is well established and several references in this field are available (Gulliman, 1953; Belevitch, 1968; Karni, 1971; Clay, 1971; Shamir, 1973; Bazaraa and Jarvis, 1977; Minieka, 1978). For consistency, the terminology used in this chapter has been adopted for pipe networks.

Also presented in this chapter are some of the fundamental hydraulic principles which form the foundation of the three traditional methods described in Chapter 3.

2.1 FUNDAMENTALS OF NETWORK THEORY

According to network terminology, a network is a graph consisting of a set of junction points called nodes, with certain pairs of nodes being joined by line segments called edges (or arcs, branches or links). Edges joining the same two nodes are multiple edges and a node without an edge connected to it is an isolated node. If a node has only one edge connected to it, that edge is a pendant. An edge and a node at the end of the edge are said to be incident. A subgraph is any collection of nodes and edges comprising only nodes and edges of a larger graph. The complement of a subgraph is the collection of nodes and edges remaining after the removal of the subgraph.

A path between two nodes is a subgraph whose terminal nodes each have only one arc incident and all other nodes are incident to exactly two arcs. A graph is said to be connected if there is a path connecting every pair of nodes. A connected subgraph in which each node of the subgraph is incident to exactly two arcs of the subgraph is called a loop (or cycle).

A tree is a connected graph containing no loops. The complement of a tree is a cotree. Edges of a cotree are links. A tree containing all nodes of a graph is a spanning tree.

An edge of a graph is said to be directed (or oriented) if there is a sense of direction ascribed to the edge. If all edges of a graph are directed, it is called a directed graph. However, a network need not be directed because it may be feasible to have flow in either direction along an edge. The flow capacity of an edge in a specified direction is the upper limit to the feasible magnitude of the rate of flow in the edge in that direction. The flow capacity may be any nonnegative quantity, including infinity. An edge is directed if the flow capacity is zero in one direction.

The topology of a directed graph of η nodes and ε edges can be described by a $\eta \times \varepsilon$ node incidence matrix, A, with typical element

$$a_{ij} = \begin{cases} +1, & \text{if edge } j \text{ is directed away from node } i \\ -1, & \text{if edge } j \text{ is directed towards node } i \\ 0, & \text{if edge } j \text{ is not connected to node } i \end{cases}$$

For a connected graph it is apparent each column of A will contain a 1 and a -1 and all remaining elements will be zero. As a check, addition of the rows of A should yield a zero row. Thus, the rank, r, of A is at most $\eta - 1$.

If loops are formed, one by one, by adding links, one at a time, to a given spanning tree, it is apparent that each time a link is added a unique loop will be created. Such a loop is called a fundamental loop. A fundamental loop set for any connected graph, containing λ loops, can be described by a $\lambda \times \varepsilon$ fundamental loop matrix, B, with typical element

$$b_{ij} = \begin{cases} +1, & \text{if edge } j \text{ is in loop } i \text{ and the direction of edge } j \text{ is clockwise, say} \\ -1, & \text{if edge } j \text{ is in loop } i \text{ and the direction of edge } j \text{ is counterclockwise} \\ 0, & \text{if edge } j \text{ is not in loop } i \end{cases}$$

By performing elementary row operations on B, an identity sub-matrix of order, λ , can be obtained, implying the rank of B is λ .

Both the node incidence matrix and the fundamental loop matrix can be used to formulate the continuity and energy (or loop) sets of equations in a computer simulator.

2.2 PIPE NETWORK CONSERVATION LAWS

Pipe network parameters are introduced to develop two conservation laws utilizing graph theory. The following notation shall be adopted for convenience. A directed network will be described by a node set, N and an edge set, E of ordered pairs of nodes. Each node $n \in N$ is associated with a unique number called the head, H_n. For an edge directed from node i to node j, an edge head loss is defined as $\Delta H_{ij} = H_i - H_j$ in which $\Delta H_{ij} = -\Delta H_{ji}$. In each edge, a flow Q_{ij} exists, positive when the edge is directed from node i to node j.

A basic law to be satisfied by the flows in a network is mass conservation,

$$\sum_{(n,j)\varepsilon E} Q_{nj} - \sum_{(i,j)\varepsilon E} Q_{in} = r_n \quad \text{all } n\varepsilon N \qquad (2.1)$$

where r_n is the requirement at node n, positive for inflows (supply) and negative for outflows (demand). Denoting the vector of Q_{ij}'s by $\bar{q}$, equation (2.1) can be rewritten as

$$A\,\bar{q} = \bar{r} \qquad (2.2)$$

where $\bar{r} = (r_1, r_2, \ldots\ldots, r_n)$.

As noted in section 2.1, A has a rank of $\eta - 1$, implying one of the rows in A is redundant and can be arbitrarily omitted. The matrix A_r, obtained by deleting one row of A, say row η, is defined as the reduced node incidence matrix. A corresponding element in $\bar{r}$ is also deleted and a demand vector, $\bar{d}$, defined as $-\bar{d} = (r_1, r_2, \ldots\ldots, r_{\eta-1})$ is introduced. Then

$$A_r\,\bar{q} = -\bar{d} \qquad (2.3)$$

It should be noted, in passing, that all the rows in A will be independent, that is, rank of A $= \eta$ if pumps and reservoirs are present in the network. However, a redundant row still exists if a junction is assumed at the reservoir or pumps.

If the nodal head is unique, as assumed, then the summation of head losses around a loop is zero. This obvious proposition is used as the basis for the second fundamental network law. Thus, if L_k is the edge set for edges in fundamental loop k, k = 1, 2, ... λ, then

$$\sum_{(i,j)\varepsilon L_k} \Delta H_{ij} = 0 \quad \text{all } k \qquad (2.4)$$

Equation (2.4) can be written as

$$B \Delta h = \bar{0} \quad (2.5)$$

if Δh is defined as the vector of ΔH_{ij}'s.

If a mesh flow vector $\bar{p} = (p_1, p_2, p_3, \ldots, p_n)$ is defined, the following relationship can be written

$$\bar{q} = B^T \bar{p} \quad (2.6)$$

Thus if to each fundamental loop a unique mesh flow is associated, the flow in any edge is a linear combination of the mesh flows for fundamental loops containing the edge in question.

If pumps and reservoirs are included in the pipe network, equation (2.5) is generalized as follows:

$$\Delta E = B \Delta h - \bar{h}_p \quad (2.7)$$

where $\bar{h}_p$ = pump head vector, ΔE = vector representing the difference in total grade between two reservoirs.

In this generalized case, a junction node is assumed at a reservoir or pump and a pseudo loop is assumed to connect 2 reservoirs.

2.3 FRICTION & MINOR LOSSES

The relation between head and discharge, that is, Δh and q completes the number of equation sets required to define the network problem. Total head loss in a pipe, H, is the sum of the line loss, h_{LP}, and minor loss, h_{LM}. The line loss expressed in terms of the discharge is given by:

$$h_{LP} = K_p Q^n \quad (2.8)$$

where K_p is a pipe constant which is a function of line length, diameter and roughness and n is an exponent. Commonly used head loss equations include the Darcy-Weisbach, Hazen-Williams and Manning equations. Perhaps the most widely used of these equations is the Hazen-Williams equation, which is,

$$\left.\begin{array}{lll} \text{English System (ES)} & Q = 1.318\ C_{HW}\ AR^{0.63}\ S^{0.54} \\ \text{S.I. Units} & Q = 0.849\ C_{HW}\ AR^{0.63}\ S^{0.54} \end{array}\right\} \quad (2.9)$$

in which C_{HW} is the Hazen-Williams roughness coefficient, S is the slope of the energy line and equals h_{LP}/L, R is the hydraulic radius defined as the cross-sectional area, A, divided by the wetted perimeter, P, and for full pipes equals D/4 (where D = diameter of pipe). Table 2.1 gives values for C_{HW} for some common materials used for pressure conduits (Jeppson, 1977).

Type of Pipe	C_{HW}
PVC pipe	150
Very smooth pipe	140
New cast iron or welded steel	130
Wood, concrete	120
Clay, new riveted steel	110
Old cast iron, brick	100
Badly corroded cast iron or steel	80

Table 2.1 : Values of Hazen-Williams Coefficient

Equations (2.9) can be written in terms of h_{LP} if Q is known. Thus,

$$\text{ES} \qquad h_{LP} = \frac{8.52 \times 10^5 L}{C_{HW}^{1.852}\ D^{4.87}}\ Q^{1.852}$$

with D in inches and L in feet.

SI $$h_{LP} = \frac{10.7\ L}{C_{HW}^{1.852}\ D^{4.87}}\ Q^{1.852}$$

Principles governing the flow of fluid as well as much experimental evidence indicates that the head loss due to added turbulence or secondary flow in the presence of fittings, valves, meters and other components in a network, will be approximately proportional to the square of the velocity or the flow rate squared. Minor losses are commonly expressed in the form

$$h_{LM} = K_M\ Q^2 \qquad (2.10)$$

in which $K_M = M/(2gA^2)$.

Nominal values of M for various common appurtenances are given in Table 2.2 (Jeppson, 1977). It is apparent from these loss coefficients that minor losses can be neglected if relatively long pipelines are analyzed. However, in short pipelines, they may represent the major losses in the system, or if a valve is partly closed, its presence has profound influence on the flow rate.

2.4 PUMPS

A number of alternative methods might be used to quantify the head, h_p, produced by a pump. In some cases a constant power input is specified. In general, the relationship between pump head, h_p and discharge, Q, can be expressed as

$$h_p = P(Q) \qquad (2.11)$$

For a constant power pump,

$$P(Q) = Z/Q \qquad (2.12)$$

DEVICE	M
Globe Valve (fully open)	10
Angle Valve (fully open)	5
Gate Valve (fully open)	0.19
Gate Valve (3/4 open)	1.0
Gate Valve (1/2 open)	5.6
Ball Check Valve (fully open)	70
Foot Valve (fully open)	15
Swing Check Valve (fully open)	2.3
Close Return Bend	2.2
Tee, Through Side Outlet	1.8
Standard Short Radius Elbow	0.9
Medium Sweep Elbow	0.8
Long Sweep Elbow	0.6
45^o Elbow	0.4

Table 2.2 : Loss Coefficients for Valves and Other Pipe Fittings

where the pump constant, $Z = 550\ HP/62.4$ and HP = useful pump horsepower.

Other functions have been suggested, and a common choice is a second order polynomial of the form

$$P(Q) = AQ^2 + BQ + H_o \qquad (2.13)$$

in which A, B and H_o are constants for a given pump and might be determined by fitting Equation (2.13) to three points taken from a pump characteristic curve.

2.5 PRESSURE REGULATING VALVES

A pressure regulating valve (abbreviated PRV) is designed to maintain a constant pressure downstream from it regardless of how large the upstream pressure is. Therefore, it is apparent that the unique relationship that exists between head and discharge for line losses, minor losses and pumps, does not exist for a PRV. Solution of pipe networks which include control elements with non-unique head discharge relationships using optimization techniques is still an active research area (Collins, Cooper, Helgason, Kennington, 1978).

Exceptions to the above occurrence are: (1) If the upstream pressure becomes less than the valve setting, and (2) if the downstream pressure exceeds the pressure setting of the valve so that if the PRV were not present, the flow would be in the opposite direction to the downstream flow direction of the valve. If the first condition occurs, the valve has no effect on flow conditions. The PRV acts as a check valve, preventing reverse flow if the second condition occurs. By preventing reverse flow, the PRV allows the pressure immediately downstream from the valve to exceed its pressure setting. Thus, PRV's can perform both functions of reducing pressures in portions of a pipe distribution system if the pressures would otherwise be excessive, and may also be used to control from which sources of supply the flow comes under various demand levels. In the latter application, the PRV acts as a check valve until the pressure is reduced to critical levels by large demands at which time additional sources of supply are drawn upon. The analysis of a pipe network containing PRV's must be capable of determining which of these conditions exist.

2.6 NODE ANALYSIS

To obtain the system of equations which contains the heads at the junctions/nodes of the network as unknowns, the $\eta - 1$ independent continuity equations are written as in Equation (2.3). The relationship between discharge and head loss is then substituted into the continuity equations yielding a set of $\eta - 1$ equations in $\eta - 1$ unknown nodal heads.

Solving for Q from the exponential formula (Equation 2.8), using double subscript notation, gives

$$Q_{ij} = (\Delta H_{ij}/K_{ij})^{1/n} \tag{2.14}$$

in which $\Delta H_{ij} = (h_{LP})_{ij} + (h_{LM})_{ij}$

and $K_{ij} = (K_p)_{ij} + (K_M)_{ij}$

Substituting Equation (2.14) into the junction continuity equations gives

$$\left[\sum \left(\frac{H_i - H_j}{K_{ij}} \right)^{1/n} \right]_{out} - \left[\sum \left(\frac{H_i - H_j}{K_{ij}} \right)^{1/n} \right]_{in} = -d \tag{2.15}$$

2.7 LOOP ANALYSIS

If the discharge in each pipe is initially considered unknown instead of the head at each junction, the number of simultaneous equations to be solved is increased from $(\eta - 1)$ to $(\eta - 1 + \lambda)$ equations. However, this increase in the number of equations is somewhat compensated by a reduction in the number of non-linear equations in the system.

The analysis of flow in networks of pipes is based on the energy and mass conservation laws discussed in section 2.2. Mathematically, the continuity equations are concisely expressed as:

$$A_r \bar{q} = - \bar{d}$$

where A_r is the reduced node incidence matrix. It is apparent that each of these continuity equations is linear.

The remaining set of equations is formed by applying the energy conservation principle and expressed in terms of the fundamental loop matrix, B, as follows:

$$B \Delta h = \bar{0}$$

which has λ independent non-linear equations.

Having solved the system of equations for the discharge in each pipe, the head losses in each pipe can be determined. From a known head or pressure at one junction, the heads and pressures at each junction throughout the network, or at any point along a pipe, can be determined by subtracting the head loss from the head at the upstream junction, and accounting for differences in elevations if this be the case.

In some problems the external flows may not be known. Rather the supply of water may be from reservoirs and/or pumps. The amount of flow from these individual sources will not only depend on demands, but also will depend upon the head losses throughout the system.

2.8 CORRECTIVE MESH FLOW ANALYSIS

This method of analysis yields the least number of equations. However, like the node analysis method, all the equations are non-linear. These equations consider a corrective mesh flow as the unknowns and as discussed in section 2.2, the system of equations to solve is written as:

$$\bar{q} = B^T \bar{p}$$

in which $\bar{p}$ is the mesh flow vector. Since there are λ fundamental loops in a network, the corrective mesh flow system of equations consists of λ equations.

This method requires an initialization of the flow in each pipe which satisfies all junction continuity equations. Since these initial flow estimates generally will not simultaneously satisfy the λ head loss equations, they must be corrected before they equal the true flow rates in the pipes. A flow rate adjustment can be added with due regard for sign, to the initially assumed flow in each pipe forming a loop of the network without violating continuity at the junctions. This fact suggests establishing λ energy equations around the λ loops of the network in which the initial flow plus the corrective mesh flow rate is used as the true flow rate in the energy equations. Upon satisfying these energy equations by finding the appropriate corrective mesh flow rates, the $\eta - 1$ continuity equations would remain satisfied as they initially were. The corrective mesh flow rates may be arbitrarily taken positive in the clockwise or counter-clockwise direction, but the sign convention must be consistent around any particular loop.

* * * * *

CHAPTER 3

NEWTON-RAPHSON METHOD

The Newton-Raphson method is an iterative scheme which starts with an estimate to the solution and repeatedly computes better estimates. Unlike other methods which converge linearly, it has "quadratic convergence". Generally if quadratic convergence occurs, fewer iterations are needed to obtain the solution with a given precision than if linear convergence occurs. In addition to rapid convergence, the Newton-Raphson method is easily incorporated into a computer algorithm.

Any of the three sets of equations defining the pipe network problem, that is equations considering (1) the flow rate in each pipe unknown, (2) the head at each junction unknown and (3) the corrective mesh flow rate around each loop unknown, may be solved by this method. An initial guess is required for the Newton-Raphson method. It is the best method to use for larger systems of equations because it requires less computer storage for a given number of equations.

3.1 APPLICATION TO NODE EQUATIONS

The iterative Newton-Raphson formula for a system of equations is,

$$\bar{x}^{(m+1)} = \bar{x}^{(m)} - D^{-1}\,\bar{F}(x^{(m)}) \qquad (3.1)$$

in which the superscripts within parentheses are not exponents but denote number of iterations. The unknown vectors $\bar{x}$ and $\bar{F}$ replace the single variable x and function F and the inverse of the Jacobian, D^{-1} replaces $1/\frac{dF}{dx}$ in the formula for solving a single equation.

Adapting Equation (3.1) to solving the set of equations with the heads as unknowns, Equation (3.1) becomes

$$\bar{H}^{(m+1)} = \bar{H}^{(m)} - \bar{D}^{-1} F(H^{(m)}) \qquad (3.2)$$

Making up the Jacobian matrix D, are individual rows consisting of derivatives of that particular function with respect to the variables making up the column headings. For the system of head equations, the Jacobian is,

$$D = \begin{bmatrix} \frac{\partial F_1}{\partial H_1}, & \frac{\partial F_1}{\partial H2} & \cdots\cdots\cdots & \frac{\partial F_1}{\partial H_J} \\ \frac{\partial F_2}{\partial H_1}, & \frac{\partial F_2}{\partial H_2} & \cdots\cdots\cdots & \frac{\partial F_2}{\partial H_J} \\ \vdots & \vdots & & \vdots \\ \frac{\partial F_J}{\partial H_1}, & \frac{\partial F_J}{\partial H_2} & \cdots\cdots\cdots & \frac{\partial F_J}{\partial H_J} \end{bmatrix}$$

where J = number of junction nodes.

The Jacobian is a symmetric matrix and an algorithm for solving a linear system of equations with a symmetric matrix may be preferred for greater computational efficiency.

3.2 APPLICATION TO CORRECTIVE MESH FLOW EQUATIONS

The Newton-Raphson method when applied to this set of equations becomes

$$P^{(m+1)} = P^{(m)} - D^{-1} F(P^{(m)}) \qquad (3.3)$$

in which the Jacobian is

$$D = \begin{bmatrix} \frac{\partial F_1}{\partial P_1}, & \frac{\partial F_1}{\partial P_2} & \cdots\cdots & \frac{\partial F_1}{\partial P_L} \\ \frac{\partial F_2}{\partial P_1}, & \frac{\partial F_2}{\partial P_2} & \cdots\cdots & \frac{\partial F_2}{\partial P_L} \\ \vdots & \vdots & & \vdots \\ \frac{\partial F_L}{\partial P_1}, & \frac{\partial F_L}{\partial P_2} & \cdots\cdots & \frac{\partial F_L}{\partial P_L} \end{bmatrix}$$

where L = number of loops and P = corrective mesh flow for each loop. The Newton-Raphson method suffers from a setback of requiring a reasonably accurate initialization; otherwise it may not converge.

When PRV's are present in a pipe network, the procedure of using identical loops for the corrective flow rates and energy equations must be altered. The reasons are (1) the head drop across a PRV cannot be expressed as a function of the P's circulating through that pipe, (2) continuity at some junctions will not be satisfied if the P's are assumed to circulate through pseudo loops from artificial reservoirs created by the PRV's to another reservoir in the network. The reason is that P in a pseudo loop would extract fluid from a junction, but not add an equal flow through another pipe joining at that junction.

Consequently, some of the loops around which the energy equations are written cannot correspond to the loops around which the corrective flow rates, P, circulate. The individual P's will thus be assumed to circulate around the real loops satisfying continuity at all junctions. The energy equations will be written around loops containing pipes or other elements such as pumps or reservoirs whose head losses are functions of the discharge through them.

* * * * *

CHAPTER 4

LINEAR METHODS

Non-linearity of the function relating head and discharge is the crux of the problem in solving a pipe network. Recall that in the loop analysis, there are $\eta + \lambda - 1$ equations of which λ number of equations describing energy conservation around loops, are non-linear. The other two analyses, namely the node analysis and corrective mesh flow rate analysis, each of which having λ energy equations written around each loop of the network; both involved non-linear equations in each of its entire system of equations. Chapter 3 has dealt with the straight-forward application of the Newton-Raphson Method to linearizing the non-linear equations associated with the latter two methods. This chapter will be devoted to other linearization techniques, some of which are variations of the Newton-Raphson Method.

4.1 GRADIENT METHOD

The gradient method, which is given extensive coverage by Wood (Wood, 1981), is derived from the first two terms of the Taylor series expansion. Any function, f(x), which is continuous, that is, differentiable, can be approximated as follows:

$$f(x) \approx f(x_o) + f'(x_o)(x-x_o) \qquad (4.1)$$

It is apparent from the right hand side of Equation (4.1) that the approximation has reduced f(x) to a linear form. However, if f is a function of more than one variable, Equation (4.1) can be generalized as follows:

$$f(x(1),x(2),\ldots) = f(x(1)_o,x(2)_o,\ldots) + \frac{\partial f}{\partial x(1)}(x(1) - x(1)_o) + \frac{\partial f}{\partial x(2)}(x(2) - x(2)_o) + \ldots \quad (4.2)$$

in which the partial derivatives are evaluated at some $x(1) = x(1)_o$, $x(2) = x(2)_o$, etc.

4.1.1 ALGORITHMS FOR THE SOLUTION OF LOOP EQUATIONS

To conform to the notation used in this chapter, Equation (2.3) which neatly describes the mass conservation (continuity) equation for each of the j* nodes in the network, is rewritten as follows:

$$\sum Q_{out} - \sum Q_{in} = Q_e \quad \text{(j equations)} \quad (4.3)$$

in which Q_e denotes the external inflow or demand at the junction node, positive for inflows.

The energy conservation equation in Equation (2.5) for fundamental loops without pumps, is now rewritten to include pumps as follows:

$$\sum h_L = \sum h_P \quad (\ell \text{ equations}) \quad (4.4)$$

where h_L = energy loss in each pipe, including minor losses; h_P = energy input by pumps; ℓ = number of fundamental loops.

For any two fixed grade (or reservoir) nodes, the energy conservation equation written around this pseudo loop is written as:

$$\Delta E = \sum h_L - \sum h_P \quad \text{(f-1 equations)} \quad (4.5)$$

in which ΔE = difference in total grade between two fixed grade nodes; f = number of pseudo loops.

If p equals the number of pipes in the network, then the mass and energy equations form a set of p simultaneous equations of which (ℓ + f - 1) equations constituting the set of energy equations are non-linear.

*for networks with reservoirs

Using Equations (2.8), (2.10), (2.11) and (4.5), the energy equations expressed in terms of the discharge, Q, are

$$\triangle E = \sum (K_P Q^n + K_M Q^2) - P(Q) \qquad (4.6)$$

It can be seen that Equation (4.4) is a special case of Equation (4.6) where $\triangle E$ is zero for a fundamental loop.

Three algorithms are presently in significant use and gradient method is employed to handle the non-linear terms in Equation (4.6).

For a single pipe section, Equation (4.6) can be written as

$$f(Q) = K_P Q^n + K_M Q^2 - P(Q) \qquad (4.7)$$

which represents the grade difference across a pipe section carrying flow Q. Substituting an estimate, Q_i, for Q, and denoting $f(Q_i)$ by H_i, Equation (4.7) becomes

$$H_i = f(Q_i) = K_P Q_i^n + K_M Q_i^2 - P(Q_i) \qquad (4.8)$$

Differentiating Equation (4.7) and setting $Q = Q_i$, gives the gradient of the function at $Q = Q_i$. Thus,

$$f'(Q_i) = nK_P Q_i^{n-1} + 2K_M Q_i - P'(Q_i)$$

Denoting $f'(Q_i)$ by G_i, thus

$$G_i = nK_P Q_i^{n-1} + 2K_M Q_i - P'(Q_i) \qquad (4.9)$$

Both the function and its gradient, evaluated at $Q = Q_i$, will be used in all three algorithms for solving loop equations.

4.1.1.1 Single Path Adjustment (P) Method

This method was first described by Hardy Cross as the "Balancing Head Method" which was limited to closed loop systems and included only line losses. The procedure is generalized and summarized as follows:

(i) An initial set of flowrates which satisfies continuity at each junction node is determined.

(ii) A flow adjustment factor is computed for each path (ℓ+f-1) to satisfy the energy equation for that path and continuity must be maintained when applying the correction factor.

(iii) Step (ii) is repeated using improved solutions until the average correction factor is within a specified limit.

Equation (4.6) is used to compute the adjustment factor for a path using gradient method to linearize the non-linear energy equations. Thus,

$$f(Q) = f(Q_i) + f'(Q_i)\,\Delta Q \qquad (4.10)$$

in which $\Delta Q = Q - Q_i$, where Q_i is the estimated discharge. Applying Equation (4.10) to Equation (4.6) and solving for ΔQ gives

$$\Delta Q = \frac{\Delta E - \sum H_i}{\sum G_i} \qquad (4.11)$$

which is the flow adjustment factor to be applied to each pipe in the path. The numerator represents the imbalance in the energy relationship due to incorrect flow-rates and this procedure reduces this to a negligible quantity. Flow adjustment is carried out for all ℓ fundamental (closed) loops and (f-1) pseudo loops in the network.

4.1.1.2 <u>Simultaneous Path Adjustment (SP) Method</u>

This algorithm is similar to the corrective mesh flow method described in Section 2.8, the only difference is that gradient method is used here instead to linearize the energy equations. It is developed to improve convergence by simultaneously adjusting the flowrate in each loop representing an energy equation. The method is summarized as follows:

(i) An initial set of flowrates which satisfy continuity at each junction node is determined.

(ii) A flow adjustment factor is simultaneously computed for each loop to satisfy the energy equations without disturbing the continuity balance.

(iii) Step (ii) is repeated using improved solutions until the flow adjustment factor is within a specified limit.

The simultaneous solution of $\ell + f - 1$ equations is required to determine the loop flow adjustment factors. Each equation includes the contribution for a particular loop as well as contributions from all other loops which have pipes common to both loops.

For loop j, the head change required to balance the energy equation is expressed in terms of the flow change in loop j (ΔQ_j) and the flow changes in adjacent loops (ΔQ_k) as follows:

$$f(Q) = f(Q_i) + \frac{\partial f}{\partial Q} \Delta Q_j + \frac{\partial f}{\partial Q} \Delta Q_k$$

$$\text{or,} \quad f(Q) = f(Q_i) + f'(Q_i) \Delta Q_j + f'(Q_i) \Delta Q_k \qquad (4.12)$$

Substituting $f(Q) = \Delta E$, $f(Q_i) = \sum H_i$, $f'(Q_i) = \sum G_i$, Equation (4.12) becomes

$$\Delta E - \sum H_i = (\sum G_i) \Delta Q_j + \sum (G_i \Delta Q_k) \qquad (4.13)$$

in which $\sum H_i$ = sum of the head changes for all pipes in loop j

$(\sum G_i) \Delta Q_j$ = sum of all gradients for the same pipes times flow change for loop j. $\sum(G_i \triangle Q_k)$ = sum of gradients for pipes common to loops j and k multiply by the flow change for loop k.

A set of simultaneous linear equations is formed in terms of flow adjustment factors for each loop representing an energy equation. The solution of these linear equations provides an improved solution for another trial until a specified convergence criterion is met.

4.1.1.3 Wood's Linear (L) Method

This method developed by Wood (Wood, 1981) involves the solution of all the basic hydraulic equations for the pipe network. However, only the energy equations need to be linearized as the continuity equations are all linear. Using gradient approximation, the energy equations are linearized in terms of an approximate flowrate, Q_i as follows:

$$f(Q) = f(Q_i) + f'(Q_i)(Q-Q_i)$$

Introducing H_i and G_i as before, the above equation becomes

$$(\sum G_i)Q = \sum(G_i Q_i - H_i) + \triangle E \qquad (4.14)$$

This relationship is employed to formulate (ℓ + f - 1) energy equations which together with the j continuity equations, form a set of p simultaneous linear equations in terms of the flowrate in each pipe. One significant advantage of this scheme is that an arbitrary set of initial flowrates, which need not satisfy continuity, can start the iteration. A flowrate based on a mean flow velocity of 4 ft/sec has been used by Wood (Wood, 1981). The solution is then used to linearized the equations and successive trials are carried out until the change in flowrates between successive trials become insignificant.

4.1.2 ALGORITHMS FOR SOLVING NODE EQUATIONS

Two methods for solving the node equations are also widely used and are described here for completeness.

4.1.2.1 Single Node Adjustment (N) Method

This method was also first described in the paper by Hardy Cross and is known as the "Balancing Flows Method". The procedure is outlined as follows:

(i) A reasonable grade is assumed for each junction node in the system. The better the initial assumptions, the fewer the required trials.

(ii) A grade adjustment factor for each junction node which tends to satisfy continuity is computed.

(iii) Step (ii) is repeated using improved solutions until a specified convergence criterion is met.

The grade adjustment factor is the change in grade at a particular node ($\triangle H$) which will result in satisfying continuity and considering the grade at adjacent nodes as fixed. For convenience, the required grade correction is expressed in terms of Q_i which is the flowrate based on the values of the grades at adjacent nodes before adjustment. Thus, using gradient approximation,

$$f(Q) = f(Q_i) + f'(Q_i).\triangle Q$$

with the usual substitution,

$$\triangle Q = \sum (1/G_i) \triangle H \qquad (4.15)$$

where $\triangle H = H - H_i$, the grade adjustment factor and $\triangle Q$ denotes the flow corrections required to satisfy continuity at nodes. From Equation (4.3),

$$\Delta Q = \sum Q_i - Q_e \tag{4.16}$$

Thus, from Equations (4.15) and (4.16),

$$\Delta H = \frac{\sum Q_i - Q_e}{\sum (1/G_i)} \tag{4.17}$$

In Equation (4.17), inflow is assumed positive. The numerator represents the unbalanced flowrate at the junction node.

Q_i, the flowrate in a pipe section prior to adjustment, is computed from

$$Q_i = (\Delta H_i/K)^{1/n}$$

in which ΔH_i = grade change based on initial assumed values of grade.

If pumps are included, the following expression is used to determine Q_i:

$$\Delta H_i = KQ_i^n - P(Q_i) \tag{4.18}$$

Equation (4.18) is solved using an approximation procedure. Adjustment of the grade for each junction node is made after each trial until a specified convergence criterion is satisfied.

4.1.2.2 Simultaneous Node Adjustment (SN) Method

This method requires the linearization of the basic pipe network node equations in terms of approximate values of the grade. If the discharge in Equation (4.3) is expressed in terms of the assumed heads, it can be written as:

$$\sum \left[\frac{H_a - H_b}{K_{ab}}\right]^{1/n} = Q_e \tag{4.19}$$

for any node, a, and b denotes an adjacent node.

Equation (4.19) can be linearized with respect to grades if the flowrates are written in terms of some initial values of the grades, H_{ai} and H_{bi}, and the corrections in these grades. The gradient method is again used to calculate the flowrate in pipe section, ab. Thus,

$$Q = Q_i + \frac{\partial Q}{\partial H_a} \triangle H_a + \frac{\partial Q}{\partial H_b} \triangle H_b \qquad (4.20)$$

in which $Q = ((H_a - H_b)/K_{ab})^{1/n}$ (4.21)

$\triangle H_a = H_a - H_{ai}$ (adjustment factor for head at node a)

$\triangle H_b = H_b - H_{bi}$ (adjustment factor for head at node b)

Substituting the partial derivatives of the flowrate expression in Equation (4.21) in Equation (4.20) and simplifying, gives

$$Q = Q_i(1 - 1/n) + \frac{Q_i^{1-n}}{nK_{ab}} (H_a - H_b) \qquad (4.22)$$

The initial value of the flowrate, Q_i, is computed based on the initial values of the grades. Thus,

$$Q_i = ((H_{ai} - H_{bi})/K_{ab})^{1/n}$$

where K_{ab} may include minor losses, if any.

Using Equation (4.22), the continuity equation for each junction node can be expressed as a linear function of the variable and fixed grades of adjacent nodes and the variable grade of junction, a.

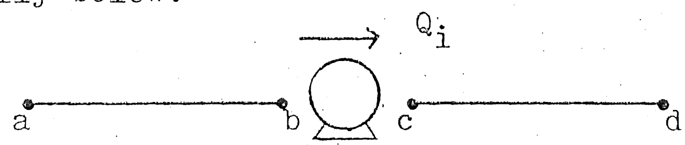
Hence,

$$\sum_{b=1}^{N_V} \frac{Q_i^{1-n}}{nK_{ab}} H_b - H_a \sum_{b=1}^{N} \frac{Q_i^{1-n}}{nK_{ab}} = \qquad (4.23)$$

$$Q_e + \sum_{b=1}^{N} Q_i + \sum_{b=1}^{N_V} \left(-\frac{Q_i}{n}\right) - H_b \sum_{b=1}^{N_F} \frac{Q_i^{1-n}}{nK_{ab}}$$

where N refers to all adjacent nodes, N_v refers to adjacent variable grade nodes and N_F refers to all adjacent fixed grade nodes. Q_i is positive for outflow.

Equation (4.23) is written for each junction node in the system resulting in a set of linear equations in terms of junction node grades. If pumps are included, two additional nodes may be assigned to a pump at the suction and discharge sides as shown schematically below:



Two additional equations can be written:

$$H_a - H_b = \frac{K_{ab}}{K_{cd}} (H_c - H_d) \qquad (4,24)$$

$$H_c - H_b = P\left[((H_c - H_d)/K_{cd})^{1/n} \right] \qquad (4.25)$$

Equation (4.24) is just the continuity equation and Equation (4.25) relates the head change across the pump to the flow in either the discharge or suction line. Equation (4.25) can be linearized using gradient method as follows:

$$\text{Let } Y = P(Q_i) + H_b - H_c = 0 \qquad (4.25a)$$

Using the gradient approximation,

$$Y = Y_i + \frac{\partial Y}{\partial H_b} \Delta H_b + \frac{\partial Y}{\partial H_c} \Delta H_c + \frac{\partial Y}{\partial H_d} \Delta H_d \qquad (4.26)$$

Substituting the partial derivatives in Equation (4.26) and simplifying, we have the following linearized equation:

$$H_c(1 + \beta) - H_b - H_d \beta = \alpha \qquad (4.27)$$

where α and β depend on the relationship used to describe the pump, P(Q), and are given by

$$\alpha = P(Q_i) - \frac{Q_i}{n} P'(Q_i)$$

$$\beta = - P'(Q_i)/(nK_{cd}Q_i^{n-1})$$

A set of $(j + 2N_p)$ simultaneous linear equations (where N_p = number of pumps) is generated and solved starting with Q_i's based on any assumed set of junction node grades. An improved set of junction node grades is then used to compute an improved set of Q_i's and the procedure repeated until a specified convergence criterion is satisfied.

4.2 COMMENTS ON ALGORITHMS USING GRADIENT METHOD

Node equations are easier to formulate because the equations include only contributions from adjacent nodes. On the other hand, the loop equations require the identification of an appropriate set of energy equations which include terms for all pipes in fundamental loops and between fixed grade nodes. Computer formulation of this set of equations is considerably more difficult than formulation of the node equations.

Each of the procedures described is iterative in nature and computations terminate when a specified convergence criterion is met. The solutions are therefore only approximate although they can be very accurate. The ability of an algorithm to produce an acceptable solution is of prime concern and studies have demonstrated that convergence problems exist and an accurate solution is not always possible.

4.2.1 ACCURACY OF SOLUTIONS

A solution is considered accurate only when all the basic equations are satisfied to a high degree of accuracy. For the three methods based on loop equations, the continuity equations are exactly satisfied. Each of these methods then proceeds to satisfy the energy equations iteratively and the unbalanced heads for the energy equations is evidence of solution accuracy. For methods based on node equations, iterations are carried out to satisfy continuity at junction nodes and the unbalance in continuity is a significant indication of solution accuracy.

4.2.2. RELIABILITY OF ALGORITHMS

A study carried out by Wood, using an extensive data base, has shown that the P, N and SN methods exhibited significant convergence problems (Wood, 1981). Since these methods are widely used, great care must be exercised when using them.

SN method failures are characterized by the inability to meet a reasonable convergence criterion and if this occurs in a limited number of trials, further trials are usually of no benefit. Failure rate was quite high and the use of results obtained employing this method is not recommended unless a good accuracy is obtained in a reasonable number of trials.

It has been established that algorithms based on node equations (N and SN methods) failed to provide reliable results because of the inability of these methods to handle low resistance lines. This is attributable to the fact that solution algorithms for these equations do not incorporate an exact continuity balance.

For each of the three methods singled out above, failure rates can be reduced if initial values closer to the correct values can be determined. However, this is no easy task and as evidenced in the study, even an excellent set of initial conditions does not guarantee convergence.

Both the SP and L methods provide excellent convergence and the attainment of a reasonable convergence criterion is sufficient to assure great accuracy. Convergence failure is very rare. However, since a gradient method is used to handle non-linear terms, there is always the possibility of convergence problems. Ill-conditioned data such as poor pump descriptions are particularly prone.

The L method has some advantages over the SP method. Assumed arbitrary flowrates need not satisfy continuity as the continuity conditions are already incorporated into the basic set of equations.

This method also allows a more straight-forward and reliable inclusion of hydraulic components such as check valves, closed lines, and pressure regulating valves. Although the SP method has significantly less equations to solve, the use of sparse matrix techniques to handle the larger matrix generated by the L method has somewhat negated this advantage.

4.3 LINEAR THEORY METHOD BY WOOD AND CHARLES

In this section, the linear theory method (Wood and Charles, 1972) will be described and used in solving the system of equations formulated by loop analysis which considers flowrates as unknowns (hereafter referred to as the Q-equations). Like the other linear method described in Section 4.1.1.3, it has several distinct advantages over the Newton-Raphson or Hardy Cross methods. Firstly, it does not require an initialization, and secondly, according to Wood and Charles, it always converges in a relatively few iterations. However, its use in solving the head oriented equations or the corrective loop oriented equations is not recommended.

Linear theory transforms the ℓ non-linear loop equations into linear equations by approximating the head in each pipe by

$$h_L = (KQ_i^{n-1})\ Q = K'Q \qquad (4.28)$$

in which Q_i is an estimate of the flowrate, and $K' = KQ_i^{n-1}$. Combining these linearized loop equations with the j-1 junction continuity equations provides a system of p linear equations which can be solved by Gaussian elimination in conjunction with sparse matrix techniques (Tewarson, 1973).

In applying the linear theory method it is not necessary to supply an initial estimate, as maybe implied. Instead, for the first iteration each K' is set equal to K, which is equivalent to setting all flowrates Q_i equal to unity. In developing the linear theory method, Wood observed that successive iterative solutions tend to oscillate about the final solution. Reasons for the oscillation can be understood by observing that the linear theory method is a variation of the Newton-Raphson method described in Chapter 3 whereby K' in Equation (4.28) is simply the derivative of h_L if multiplied by n. The oscillation could be prevented by multiplying each K by its n, which involves more computation than averaging consecutive solutions as proposed by Wood. Thus, the flowrate used in a trial is just the average flowrate for that pipe from the previous two solutions, or

$$Q_i(m) = \left[Q_i(m-1) + Q_i(m-2) \right] / 2$$

in which m within parentheses denotes a trial number.

4.3.1 INCLUSION OF PUMPS AND RESERVOIRS

When pumps (not booster pumps) and reservoirs are connected to a network, the flows in the two connected pipes become additional unknowns and therefore an additional equation is required beyond the j continuity equations and ℓ fundamental loop equations. The additional equation is obtained from a pseudo loop, which connects the two reservoirs (fixed grade nodes) by a "no flow" pipe. If f fixed grade nodes exist in a network, there would be f-1 independent equations. Energy conservation around a pseudo loop (of which a fundamental loop is a special case) is defined by Equation (4.6). Thus,

$$\Delta E = \sum (K_p Q^n + K_M Q^2) - P(Q)$$

If the expression for P(Q) in Equation (2.13) is adopted in Equation (4.6), the linear theory method does not give rapid convergence as it does when pumps and/or reservoirs are not present. A modification will therefore have to be made to allow the linear theory method to converge rapidly. The reason for the modification is that the head produced by a typical centrifugal pump decreases nearly proportional to the reciprocal of the square root of the flowrate whereas the head loss in a typical pipe increases approximately proportional to the square of the flowrate. A consequence of using this typical pump relationship in Equation (4.6) is that if the equation is solved by the linear theory method, convergence may become very slow if at all.

This situation can be improved by a transformation of variables so that the new unknown has an exponent close to n. Such a transformation is

$$G = Q + B/2A \qquad (4.29)$$

in which G is the new variable and A and B are the same constants in Equation (2.13). The appropriateness of Equation (4.29) is demonstrated by solving it for Q and substituting in Equation (2.13). After some simplification,

$$h_p = AG^2 + h_o \qquad (4.30)$$

where $$h_o = H_o - B^2/4A$$

Obviously, the exponent of G (that is, 2) is close to the typical n. Substituting Equation (4.30) in Equation (4.6) gives

$$\sum(K_p Q^n + K_M Q^2) - \sum AG^2 = \Delta E + \sum h_o \qquad (4.31)$$

Addition of Equations (4.29) and (4.31) produces a system with as many equations as unknowns.

4.3.2 INCLUSION OF PRESSURE REGULATING VALVES (PRV'S)

Networks containing PRV's may be analyzed by the linear theory method by initially assuming that the pressure (or head) immediately downstream from a PRV is constant and equal to the valve setting. Junction continuity equations are then written as if no PRV's are present. To write the loop equations, pipes containing PRV's are disconnected from the upstream nodes and the PRV's are replaced by dummy reservoirs. After each iteration a check on the flowrate Q, in each pipe containing a PRV is made. If there is any negative Q, the pseudo loop equation which includes terms for that pipe is modified with Q replaced by an unknown grade (head) immediately downstream from that PRV.

* * * * *

CHAPTER 5

ALTERNATIVE MATHEMATICAL APPROACHES & COMPUTATIONAL EXPERIENCE

5.1 MATHEMATICAL PROGRAMMING TECHNIQUES

As a prelude to introducing the alternative approaches to solving the governing equations for a pipe network using optimization techniques, it is convenient to define a network topology using notations which are consistent with those used in graph theory. Let the network topology be described by a node set N and an arc set (network element) E_o. In each of the set E_o, let Q_{ij} denote the flowrate from node i to j. Each node, n in the set N is associated with a hydraulic head, H_n. Let R, a subset of N, be the set of nodes corresponding to reservoirs (fixed grade nodes) and let H_n^* for all $n \in R$ be the fixed head associated with a reservoir. Also let r_n for all $n \in N$ denote the flow requirements (that is, supply or demand) at node n. For an incompressible fluid, the governing network equations can be stated as:

$$\sum_{(n,j)\in E} Q_{nj} - \sum_{(i,n)\in E} Q_{in} = r_n , \quad \text{all } n \in N \qquad (5.1)$$

$$\sum_{n\in N} r_n = 0 \qquad (5.2)$$

$$H_i - H_j = F_{ij}(Q_{ij}), \quad \text{all } (i,j)\in E_o \qquad (5.3)$$

$$H_n = H_n^* , \quad \text{all } n \in R \qquad (5.4)$$

Equation (5.1) is just a statement of mass conservation at each node while Equation (5.2) stipulates mass conservation for the network as a whole. Equation (5.3) states that the head loss $H_i - H_j = \Delta H_{ij}$

across an element is some function F_{ij} of the discharge through the element while Equation (5.4) requires that at a reservoir node, the head is constant. The functional form of F_{ij} or its inverse E_{ij} $(\Delta H_{ij}) = Q_{ij}$ is not specified and can represent any element including simple pipes and minor loss devices as long as a unique relationship between head and discharge exists.

In general, F_{ij} for most or all (i,j) in E_o is non-linear, thus necessitating iterative techniques such as (i) Hardy Cross, (ii) Newton-Raphson, and (iii) linearization, to be used to solve the governing network equations. Most of these techniques are detailed in Chapters 3 and 4. Each of these methods is simply a technique for solving a set of non-linear simultaneous equations which have been adapted to the network analysis problem. Each is iterative in nature and begins with an initial trial solution. A new solution is obtained by solving a set of linear equations using straightforward procedures. If the new solution differs from the trial solution by less than a specified amount then the iteration stops. Otherwise, the new solution becomes the trial solution and the procedure is repeated. In some of the algorithms, an initial trial solution sufficiently close to the true solution is required to ensure convergence. The differences in the methods result from the use of different strategies to determine the new solution.

The new approach by Collins, Cooper, Helgason and Kennington (1978) represents a radical departure from the state of the art iterative methods as optimization models are employed to solve the network problem. Two alternatives models are formulated and these models play analogous roles to the node versus loop formulations for solution of the network equations used in the state of the art methods.

The first of the two optimization models, called the Content Model assumes the form

$$\text{Minimize } G = \sum_{(i,j)\in E}\left[\int_0^{Q_{ij}} F_{ij}(t)dt\right] - \sum_{(g,n)\in E_1}\left[\int_0^{Q_{gn}} H_n^* dt\right] + \sum_{(n,g)\in E_1}\left[\int_0^{Q_{ng}} H_n^* \, dt\right]$$

Subject to

$$\sum_{(n,j)\in EUE_1} Q_{nj} - \sum_{(i,n)\in EUE_1} Q_{in} = r_n \text{ , all } n \in NU(g)$$

$$Q_{ij} \geq 0, \text{ all } (i,j) \in E_0 U(E_1)$$

in which E is the arc set for a network in which the arcs have been replaced by two equivalent oppositely directed one-way elements so that Q_{ij} can assume only positive values. This replacement is done as a mathematical convenience so that Q_{ij} can be treated as a constrained decision variable in the optimization model which has a non-negativity condition imposed on Q_{ij}. It can be proven that the solution obtained by solving the E network will produce identical results as those which would be obtained by solving the original E_0 network which permits Q_{ij} to be unconstrained. The arc set E_1 is merely a set of arcs connecting all nodes in N to a ground node g and is introduced to satisfy mass conservation for the network as a whole (Equation (5.2)).

Using the terminology of Cherry and Millar (1951), the above problem is to find a set of flows which satisfies flow conservation and minimizes system content, G, hence the name Content Model.

The second optimization model, the Co-Content Model, is a complementary (but not dual) model which has the form

$$\text{Minimize} \quad J = \sum_{(i,j)\in E} \left[\int_0^{\Delta H_{ij}} E_{ij}(t)\,dt \right] - \sum_{n\in N} \left[\int_0^{\Delta H_{ng}} r_n \, dt \right]$$

subject to

$$\Delta H_{ij} + \Delta H_{jg} - \Delta H_{ig} = 0, \text{ all } (i,j) \in E$$

$$\Delta H_{ng} = H^*_n - H^*_g \text{ , all } n \in R$$

In the terminology of Cherry and Millar (1951), the above problem is to find a set of head losses which sums to zero around all loops and minimizes system Co-Content, J, hence the name Co-Content Model.

Using Kuhn-Tucker theory (Kuhn and Tucker, 1950), it can be proved that the solution to either of these models yields the solution to the pipe network problem, that is, the optimal solution satisfies the governing network equations. The proof is carried out by examining the derivatives of the objective function and showing that the derivative conditions for a stationary point, along with other constraints, are identical to the network equations.

In the proof, it is assumed that the F_{ij} and E_{ij} functions are monotonically increasing. This assumption insures the convexity of the objective function which in turn guarantees the existence of a unique solution to the optimization problem. The monotonicity of F_{ij} and E_{ij} merely implies the fact that energy losses in a network element increase with increasing discharge.

The Content Model has the special structure of a convex cost network flow problem for which efficient routines are available. Numerous non-linear algorithms such as (i) Frank-Wolfe method,

(ii) piece-wise linear approximation and (iii) convex-simplex method are available for solving such a problem.

The use of mathematical programming techniques in pipe network analysis has paved the way for potential research in the following areas:

(i) Extension of mathematical programming techniques to solution of compressible flow pipe network analysis problem.

(ii) Incorporation of time variable storage in network elements to solve transient network problems.

(iii) Use of mathematical programming techniques to solve complex open channel networks.

(iv) Feasibility of using mathematical programming techniques to solve network parameter identification problems such as the head-discharge relationship in pipe network analysis.

(v) Development of an economic model to minimize the operational costs for a flow network with operational behavior given by one or more network problems.

5.2 COMPUTATIONAL EXPERIENCE

A computer program was written based on the linear theory method (Wood and Charles, 1972) described in Section 4.3. The program was designed to solve the system of loop and node equations using the iterative procedure described by the method. Two features this FORTRAN computer program may have for general application include:

(i) the capability of handling networks containing pumps and reservoirs, and

(ii) an algorithm which analyzes networks containing pressure regulating valves.

The use of the node incidence matrix and fundamental loop matrix described in Section 2.1 in the algorithm has provided an efficient means of translating information contained in any pipe network into a network simulator. Incidentally, the node equations and loop equations were formulated using the node incidence matrix and fundamental loop matrix respectively.

In carrying out all computations, friction losses in pipes were assumed to be described by the Hazen-Williams equation and pumps were described by the quadratic form (Equation 2.13). The convergence criterion employed was:

$$\frac{\sum \left| Q_i - Q_{i-1} \right|}{\sum \left| Q_i \right|} \leq 0.0005$$

in which Q_i is the flowrate obtained for a trial and Q_{i-1} is the flowrate obtained from the preceding trial. This appears to be a stringent requirement which may assure good accuracy if the condition is satisfied. However accuracy is achieved if and only if continuity at every node and the energy equations are exactly satisfied.

A small scale network, taken from Jeppson (1977, p.109) and shown in Fig 5.1, was tested. This 8 pipe, 5 node network, with the properties given in Table 5.1, has 2 reservoirs, a pump and a pressure regulating valve. The solution to the test problem and the solution reported by Jeppson (1977, p.110), using the same theory are tabulated in Table 5.2. The results appear to be in good agreement.

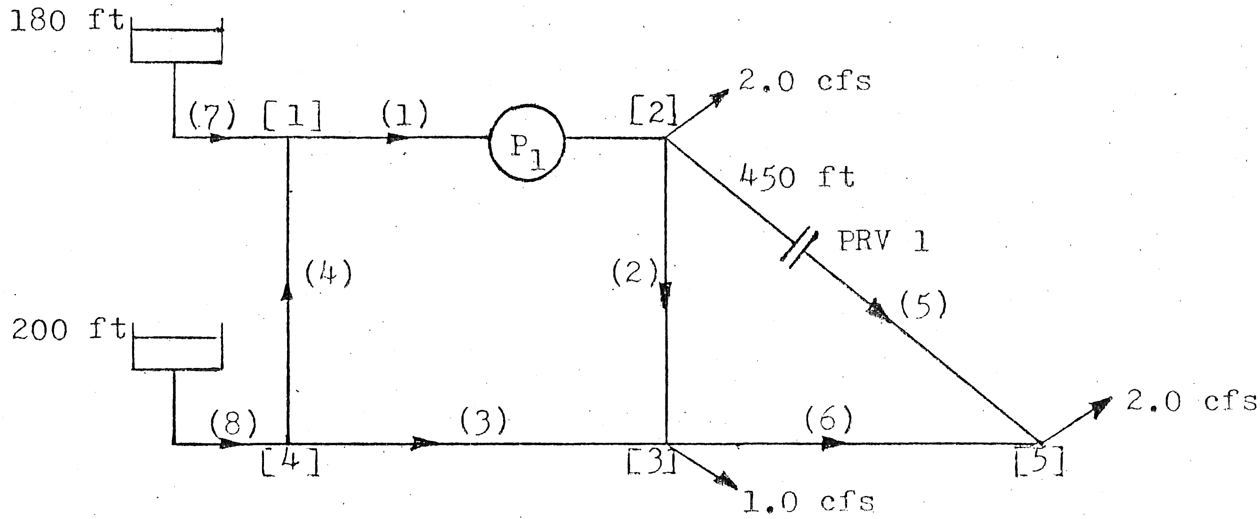


Fig 5.1 - Test Problem

Pipe	Length (ft)	Diameter (in)	Hazen-Williams Coefficient
1	1000	6	110
2	800	6	120
3	1000	6	110
4	800	6	120
5	1200	6	120
6	1000	6	120
7	500	8	130
8	500	8	130

Table 5.1 - Network Parameters

Discharge (cfs)	Head (ft)
1.0	40.0
1.5	35.0
2.0	26.0

Pump Characteristics

Pipe	Discharge, cubic ft per sec	
	Writer's Solution	Jeppson's Solution
1	2.53	2.56
2	-0.38	-0.32
3	2.47	2.44
4	0.72	0.73
5	0.92	0.88
6	1.08	1.12
7	1.81	1.83
8	3.19	3.17

Table 5.2 - Solution to Test Problem

The detailed solution and program listing are contained in the Appendix. With this program, the test problem took 0.12 second of execution time on an Amdahl 470 computer. The number of iterations required to meet the convergence criterion was 6. The subroutine used for solving the linearized set of loop equations and the linear node equations simultaneously was developed based on the Gaussian method of elimination improved by pivotal condensation (Tewarson, 1973).

The capability of the program to handle a larger network has not been proven but it would have stretched the available storage of a computer to its limits if it has been tested. Storage space is primarily taken up by the final augmented matrix which comprises essentially the node and loop equations. The use of sparse matrix techniques instead of full matrix methods may extend the capability of the program to analyze larger networks of a few hundred pipes and nodes.

* * * * *

CHAPTER 6

CONCLUSION

The Hardy Cross method which sparked off the evolution of the numerous techniques of simulating pipe networks, is suitable only for relatively small networks. With the advent of the computer, and as larger and more complex networks were analyzed, the Hardy Cross method was found to frequently converge too slowly if at all. The classic method which is described in most hydraulics or fluid mechanics text books, is an adaptation of the Newton-Raphson method which solves one equation at a time before proceeding to the next equation during each iteration instead of solving all equations simultaneously. The single path and single node methods described in Sections 4.1.1.1 and 4.1.2.1 respectively, are basically the classic Hardy Cross methods. Procedures developed to improve the convergence of the single path method were described by Martin and Peters (1963) and later by Epp and Fowler (1970). The procedure involves the simultaneous computations of flow adjustments and was presented in Section 4.1.1.2. A similar approach has been developed for the node equations where all node equations are linearized and solved simultaneously. This method is described by Shamir and Howard (1968). All of the four methods mentioned so far require an initial guess as to the solution and the rate of convergence depends to a degree on how close this initialization is to the correct solution.

For the system of equations which is flowrate oriented, two linearization techniques (Wood, 1981 and Wood and Charles, 1972) were described in Sections 4.1.1.3 and 4.3 respectively. Both of these procedures do not require an initialization and have been reported to converge in a relatively few iterations.

Significant convergence problems were reported for the Single Path, Single Node and Simultaneous Node Methods (Wood, 1981). It has been suggested that if a specified stringent convergence criterion cannot be met using single adjustment methods, the solution is probably unreliable. For the simultaneous node adjustment method, it has been suggested that the best indication of an acceptable solution is that the average relative unbalanced flow at the junction nodes be less than 2%. Instances of failures have also been reported in cases where line losses vary greatly or pumps operate on steep curves even when good initial approximations are available.

The simultaneous path methods and the linear method using gradient approximations, were reported to provide excellent convergence and the attainment of a stringent convergence criterion is sufficient to assure great accuracy in most cases. In the study carried out by Wood (1981), in which a wide variety of situations was represented, some incorporating features which increase convergence difficulties like low resistance lines; these methods were reported to attain accurate solutions in a relatively few iterations. However, if gradient approximations are used to handle non-linearity, convergence problems are always a possibility, especially if ill-conditioned data such as poor pump descriptions are employed.

Of all methods, the linear methods developed by Wood and Charles (1972) and a later version by Wood (1981), who used gradient approximations, offer more advantages. A balanced initial set of flowrates is not required since the continuity conditions are already incorporated into the basic set of equations. These algorithms permit a more direct and reliable incorporation of hydraulic components such as check valves, closed lines and pressure regulating valves. For any pipe network simulators to be of general use, these components, which affect continuity, and their effects on the hydraulics of the network must be incorporated into the basic set of equations. However, the set of equations solved by the linear methods involved significantly more equations which will be a setback if full matrix methods are used. The use of sparse matrix techniques has somewhat corrected this disadvantage and has rendered it a more desirable algorithm to adopt for analysis of pipe networks.

The use of mathematical programming techniques in pipe network analysis holds a lot of promise for the future. One of the direct consequences of the theory described in Section 5.1 is the identification of a unitary measure by which the goodness of a solution can be gaged. Traditional methods described previously give no good insight into the goodness of an approximate solution, particularly for large scale problems. The optimization models remove the vagueness that inherently surrounds a definition of " close " when an attempt is made to utilize a comparison of individual flows, heads, or losses in individual elements. Optimization techniques also have their setbacks. One is that functions describing friction losses, minor losses in pipes and pump heads must necessarily be convex functions for a solution to be guaranteed. In addition, head loss must be a unique function of discharge. Such uniqueness may not exist for certain control elements such as check valves and pressure regulating valves. Until these problems are resolved, its application will be limited in scope.

* * * * *

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APPENDIX


```
$JOB
      DIMENSION HWC(100),XL(100),DIA(100),XKM(100),RFLOW(60),
     *JN(60,10),NN(60),NVALV(10),KP(10),A(10),B(10),HO(10),
     *LP(40,20),NLP(40),H2(40),H1(40),Q(100,12),G(10,12),
     *FIM(110,111),XKT(100),X(100),KPRVS(10),HGL(10),
     *JAC(10),HP(10,3),QP(10,3),JNL(40,20),NIK(40),NBJ(10),
     *XLPRV(10),HD(60),HGLU(10),VALST(10)
C     NOTE: FOLLOWING DATA ARE FORMATTED (7I3)
C     NP=NO. OF PIPES (MAX=100,COL 1-3);NJ=NO. OF NODES (MAX=60,
C     COL 4-6);NL=TOTAL NO. OF LOOPS (MAX=40,COL 7-9);NPUMP=NO. OF
C     PUMPS (MAX=10,COL 10-12); NRLP=NO. OF REAL LOOPS(LOOPS CON-
C     TAINING NO RESERVOIRS/PRESSURE REGULATING VALVES(PRV'S),COL
C     13-15);NPRV=NO. OF PRV'S(MAX=10,COL 16-18);MAX=NO. OF ITERA-
C     TIONS (DEFAULT=8, MAX=12,COL 19-21)
      READ 1, NP,NJ,NL,NPUMP,NRLP,NPRV,MAX
    1 FORMAT (7I3)
      IF (MAX) 3,3,2
    3 MAX=8
    2 DO 5 I=1,NP
C     X(L)=LENGTH OF PIPE 'I' IN FT (FOR PIPES WITH PRV'S,DOWN-
C     STREAM LENGTH IS READ INSTEAD);DIA(I)=DIAMETER OF PIPE 'I'
C     IN INCHES;HWC(I)=HAZEN-WILLIAMS COEFFICIENTS;XKM(I)=MINOR
C     LOSS COEFFICIENTS FOR VALVES, BENDS & OTHER FITTINGS IN PIPE
C     'I'.
    5 READ, XL(I), DIA(I), HWC(I), XKM(I)
      DO 7 I=1,NJ
C     RFLOW(I)=DEMAND(-VE)/SUPPLY(+VE) AT NODE 'I';NNJ=NO. OF PIPES
C     MEETING AT NODE 'I' (MAX=10);JN(I,J)=I/D NUMBERS OF PIPES
C     MEETING AT NODE 'I' (ANY ORDER)
C     AT NODE 'I' (CAN BE ARRANGED IN ANY ORDER)
      READ, RFLOW(I),NNJ,(JN(I,J),J=1,NNJ)
    7 NN(I)=NNJ
      DO 1000 I=1,NPRV
C     NVALV(I)=I/D NUMBERS OF PIPES WITH PRV'S (FOR IDENTIFICATION,PRV'S
C     TO BE NUMBERED CONSECUTIVELY FROM 1 & NUMBERS OF PIPES  READ IN
C     SAME ORDER AS PRV'S ARE CONSECUTIVELY NUMBERED)
C     NBJ(I)=NODE UPSTREAM OF PRV; XLPRV(I)=PIPE LENGTH UPSTREAM
C     OF PRV (FT); VALST(I)=PRV SETTING (FT)
 1000 READ, NVALV(I), NBJ(I), XLPRV(I), VALST(I)
      DO 6 I=1,NPUMP
C     NOTE: FOLLOWING DATA ARE FORMATTED (I3,3F8.3)
C     KP(I)=NUMBER ASSIGNED TO PIPE 'I' WHICH CONTAINS A PUMP (COL 1-3);
C     A(I)(COL 4-11),B(I)(COL 12-19),HO(I)(COL 20-27)=PUMP CONSTANTS IN
C     THE EQUATION, PUMP HEAD,HP=A*Q**2+B*Q+HO (COLS ASSIGNED TO A,B &
C     HO TO BE LEFT BLANK IF HP'S & QP'S ARE SPECIFIED LATER IN PROGRAM).
      READ 11, KP(I), A(I), B(I), HO(I)
   11 FORMAT (I3,3F8.3)
      IF (A(I)) 6,9,6
    9 DO 15 J=1,3
C     HP(I,J)=PUMP HEAD IN FT. CORRESPONDING TO PUMP IN PIPE NO. KP(I);
C     QP(I,J)=CORRESPONDING DISCHARGE IN CFS (3 SETS OF HP & QP TO BE
C     SPECIFIED, EACH TO A LINE)(NEED NOT BE SPECIFIED IF A,B & HO
C     ARE SPECIFIED EARLIER IN PROGRAM).
      READ, HP(I,J),QP(I,J)
      FIM(J,1)=QP(I,J)**2.
      FIM(J,2)=QP(I,J)
      FIM(J,3)=1.
   15 FIM(J,4)=HP(I,J)
      CALL GAUSS(FIM,X,3)
      A(I)=X(1)
      B(I)=X(2)
      HO(I)=X(3)
    6 CONTINUE
      NAPC=NP+NPUMP+1
      NJLR=NJ+NL+NPUMP
      DO 8 I=1,NL
      IF (I-NRLP) 10,10,12
C     NNLP=NO. OF PIPES FORMING A REAL LOOP (MAX=20);LP(I,J)=NUMBERS OF
C     PIPES IN REAL LOOP 'I', ARRANGED IN CLOCKWISE ORDER, +VE IF
C     CLOCKWISE & -VE, OTHERWISE.
   10 READ, NNLP,(LP(I,J),J=1,NNLP)
      GO TO 14
C     NNLP=NO. OF PIPES FORMING A PSEUDO LOOP (LOOPS CONTAINING
C     RESERVOIRS/PRV'S, MAX=20);LP(I,J)=I/D NOS. OF PIPES IN PSEUDO
C     LOOP 'I' ARRANGED IN CLOCKWISE ORDER, +VE IF CLOCKWISE, & -VE,
C     OTHERWISE; H2,H1=RESERVOIR/PRV HEADS IN LOOP 'I',ARRANGED IN
C     CLOCKWISE ORDER ALONG PATH CONNECTING RESERVOIRS/PRV'S.
   12 READ, H2(I-NRLP),H1(I-NRLP),NNLP,(LP(I,J),
     *  J=1,NNLP)
   14 NLP(I)=NNLP
    8 CONTINUE
      DO 1100 I=1,NL
C     NNJN=NO. OF NODES IN LOOP (MAX=20);JNL(I,J)=I/D NOS. OF NODES
C     IN LOOP ARRANGED IN SAME ORDER AS PIPES IN THE LOOP WITH EACH
```

```
C     NODE I/D NO. READ IN AFTER A PRECEDING PIPE I/D NO.
      READ, NNJN, (JNL(I,J),J=1,NNJN)
 1100 NIK(I)=NNJN
      LX=0
   16 DO 18 I=1,NP
   18 Q(I,1)=1.
      DO 20 I=1,NPUMP
   20 G(I,1)=1.
      KB=1
  500 KB=KB+1
      DO 25 I=1,NJLR
      DO 25 J=1,NAPC
   25 FIM(I,J)=0.
C     FORMULATION OF NODE INCIDENCE MATRIX
      DO 30 I=1,NJ
      NM=NN(I)
      DO 30 J=1,NM
      MN=IABS(JN(I,J))
      IF (JN(I,J)) 26,28,28
   26 FIM(I,MN)=-1.
      GO TO 30
   28 FIM(I,MN)=1.
   30 CONTINUE
C     FORMULATION OF FUNDAMENTAL LOOP MATRIX
      DO 40 I=1,NL
      MN=NLP(I)
      DO 40 J=1,MN
      ML=IABS(LP(I,J))
      IF (LP(I,J)) 33,37,37
   33 FIM(NJ+I,ML)=-1.
      GO TO 40
   37 FIM(NJ+I,ML)=1.
   40 CONTINUE
      DO 60 I=1,NL
      IF (I-NRLP) 44,44,42
   42 FIM(NJ+I,NAPC)=H2(I-NRLP)-H1(I-NRLP)
   44 LN=NLP(I)
      DO 62 J=1,LN
      KPA=IABS(LP(I,J))
      DO 61 K=1,NPUMP
      IF (KPA-KP(K)) 61,46,61
   46 IF (LP(I,J)) 48,50,50
   48 HPS=B(K)**2./(4.*A(K))-HO(K)
   49 FIM(NJ+I,NAPC)=FIM(NJ+I,NAPC)+HPS
      GO TO 62
   50 HPS=HO(K)-B(K)**2./(4.*A(K))
      GO TO 49
   61 CONTINUE
   62 CONTINUE
   60 CONTINUE
      DO 65 I=1,NJ
   65 FIM(I,NAPC)=RFLOW(I)
C     COMPUTATION OF LINE & MINOR LOSSES
      DO 70 I=1,NP
   70 XKT(I)=(8.52E5*XL(I)*(ABS(Q(I,KB-1)))**0.852)
     */(HWC(I)**1.852*DIA(I)**4.87)+8.*XKM(I)*ABS(
     *Q(I,KB-1))/(32.2*3.141593**2.*DIA(I)**4.)
      DO 75 I=1,NL
      DO 75 J=1,NP
   75 FIM(NJ+I,J)=FIM(NJ+I,J)*XKT(J)
      NJL=NJ+NL
      DO 100 I=1,NL
      LM=NLP(I)
      DO 100 J=1,LM
      KPA=IABS(LP(I,J))
      DO 100 K=1,NPUMP
      IF (KPA-KP(K)) 100,80,100
   80 IF (LP(I,J)) 82,84,84
   82 FIM(NJ+I,NP+K)=A(K)*ABS(G(K,KB-1))
      GO TO 86
   84 FIM(NJ+I,NP+K)=-A(K)*ABS(G(K,KB-1))
   86 FIM(NJL+K,KPA)=-1.
      FIM(NJL+K,NP+K)=1.
      FIM(NJL+K,NAPC)=B(K)/(2.*A(K))
  100 CONTINUE
      IF  (LX) 102,102,200
  102 CALL GAUSS(FIM,X,NJLR)
      QTOT=0.
      QFLCH=0.
      DO 105 I=1,NP
      Q(I,KB)=X(I)
      QTOT=QTOT+ABS(X(I))
  105 QFLCH=ABS(Q(I,KB)-Q(I,KB-1))+QFLCH
      ERR=QFLCH/QTOT-0.0005
```

```
      DO 108 I=1,NPUMP
  108 G(I,KB)=X(NP+I)
      IF (ERR) 110,110,130
  110 DO 115 I=1,NPRV
      KPRV=NVALV(I)
      IF (Q(KPRV,KB)) 112,115,115
  112 LX=1
      KKB=KB
      GO TO 16
  115 CONTINUE
      GO TO 900
  130 IF (KB-MAX) 140,140,150
  140 IF (KB-2) 500,500,141
  141 DO 142 I=1,NP
  142 Q(I,KB)=(Q(I,KB)+Q(I,KB-1))/2.
      DO 145 I=1,NPUMP
  145 G(I,KB)=(G(I,KB)+G(I,KB-1))/2.
      GO TO 500
  150 PRINT 155
  155 FORMAT (2X,'DESIRED ACCURACY CANNOT BE ATTAINED')
      PRINT 160
  160 FORMAT (2X,'IN NO. OF ITERATIONS SPECIFIED'/)
      PRINT 165, MAX
  165 FORMAT (2X,'NO. OF ITERATIONS SPECIFIED = ',I2/)
      PRINT 170, ERR
  170 FORMAT (2X,'ERROR = ',F10.5//)
      GO TO 900
  200 LX=0
      DO 250 I=1,NPRV
      KPRV=NVALV(I)
      IF (Q(KPRV,KKB)) 210,250,250
  210 LX=LX+1
      KPRVS(LX)=KPRV
      NRLP1=NRLP+1
      DO 240 J=NRLP1,NL
      LJ=NLP(J)
      DO 245 JJ=1,LJ
      IF (KPRV-IABS(LP(J,JJ))) 245,213,245
  213 IF (JJ-NLP(J)) 215,217,217
  215 FIM(NJ+J,NAPC)=FIM(NJ+J,NAPC)-H2(J-NRLP)
      FIM(NJ+J,KPRV)=-1.
      GO TO 245
  217 FIM(NJ+J,NAPC)=FIM(NJ+J,NAPC)+H1(J-NRLP)
      FIM(NJ+J,KPRV)=1.
  245 CONTINUE
  240 CONTINUE
  250 CONTINUE
      CALL GAUSS(FIM,X,NJLR)
      DO 300 I=1,LX
      KNO=KPRVS(I)
      DO 300 J=1,NPRV
      IF (KNO-NVALV(J)) 300,270,300
  270 HGL(J)=X(KNO)
      X(KNO)=0.
      JAC(I)=J
  300 CONTINUE
      QTOT=0.
      QFLCH=0.
      DO 310 I=1,NP
      Q(I,KB)=X(I)
      QTOT=QTOT+ABS(X(I))
  310 QFLCH=ABS(Q(I,KB)-Q(I,KB-1))+QFLCH
      ERR=QFLCH/QTOT-0.0005
      IF (ERR) 320,320,350
  320 DO 327 I=1,LX
      PRINT 325
  325 FORMAT (2X,'HYDRAULIC GRADE IMMEDIATELY')
      PRINT 330, JAC(I), HGL(I)
  330 FORMAT (2X,'DOWNSTREAM OF PRV',I3,' = ',F6.2/)
  327 CONTINUE
      GO TO 900
  350 IF (KB-MAX) 360,360,400
  360 IF (KB-2) 500,500,361
  361 DO 380 I=1,NP
      DO 365 J=1,LX
      IF (I-KPRVS(J)) 365,370,365
  365 CONTINUE
      Q(I,KB)=(Q(I,KB)+Q(I,KB-1))/2.
      GO TO 380
  370 Q(I,KB)=0.
  380 CONTINUE
      GO TO 500
  400 PRINT 410
  410 FORMAT (2X,'DESIRED ACCURACY CANNOT BE ATTAINED')
```

```
      PRINT 420
  420 FORMAT (2X,'IN NO. OF ITERATIONS SPECIFIED')
      PRINT 430, MAX
  430 FORMAT (2X,'NO. OF ITERATIONS SPECIFIED = ',I2)
      PRINT 440, ERR
  440 FORMAT (2X,'ERROR = ',F8.5/)
      GO TO 320
  900 PRINT 890
  890 FORMAT ('1',15X,'P I P E  D I S C H A R G E')
      PRINT 892
  892 FORMAT (16X,'***************************'/)
      DO 910 I=1,NP
      PRINT 920, I, Q(I,KB)
  920 FORMAT (16X,'DISCHARGE IN PIPE NO. ',I3,' = ',F5.2,
     *'  CFS'/)
  910 CONTINUE
      DO 2000 I=1,NP
      DO 1500 J=1,NPRV
      IF (I-NVALV(J)) 1500,1400,1500
 1400 XL(I)=XL(I)+XLPRV(J)
      GO TO 1450
 1500 CONTINUE
 1450 XKT(I)=(8.52E5*XL(I)*(ABS(Q(I,KB)))**1.852)/
     *(HWC(I)**1.852*DIA(I)**4.87)+8.*XKM(I)*ABS(Q(I,KB))/
     *(32.2*3.141593**2.*DIA(I)**4.)
 2000 CONTINUE
      PRINT 4100
 4100 FORMAT (///16X,'H E A D  L O S S E S')
      PRINT 4102
 4102 FORMAT (16X,'*********************'/)
      DO 4300 I=1,NP
      PRINT 4320, I, XKT(I)
 4320 FORMAT (16X,'HEAD LOSS IN PIPE NO. ',I3,' = ',F8.2
     *,'  FT'/)
 4300 CONTINUE
      DO 2500 I=1,NL
      MZ=NLP(I)
      DO 2400 J=1,MZ
      MZP=IABS(LP(I,J))
      IF (Q(MZP,KB)) 2410,2400,2400
 2410 LP(I,J)=-LP(I,J)
 2400 CONTINUE
 2500 CONTINUE
      DO 3002 J=1,NJ
 3002 HD(J)=0.
      NRLP1=NRLP+1
      DO 3000 I=NRLP1,NL
      NT=NIK(I)
      DO 3100 J=1,NT
      JP=JNL(I,J)
      KIP=IABS(LP(I,J))
      DO 3200 K=1,NPUMP
      IF (KIP-KP(K)) 3200,3150,3200
 3150 HP1=A(K)*(ABS(Q(KIP,KB)))**2.+B(K)*
     *ABS(Q(KIP,KB))+HO(K)
      GO TO 3210
 3200 CONTINUE
      HP1=0.
 3210 IF (J-1) 3212,3212,3240
 3212 DO 3218 L=1,NPRV
      IF (KIP-NVALV(L)) 3218,3214,3218
 3214 XKT(KIP)=XKT(KIP)*(XL(KIP)-XLPRV(L))/XL(KIP)
      GO TO 3220
 3218 CONTINUE
 3220 IF (LP(I,J)) 3225,3230,3230
 3225 HD(JP)=H2(I-NRLP)+XKT(KIP)+HP1
      GO TO 3100
 3230 HD(JP)=H2(I-NRLP)-XKT(KIP)+HP1
      GO TO 3100
 3240 JP1=JNL(I,J-1)
      IF (LP(I,J)) 3245,3247,3247
 3245 HD(JP)=HD(JP1)+XKT(KIP)+HP1
      GO TO 3100
 3247 HD(JP)=HD(JP1)-XKT(KIP)+HP1
 3100 CONTINUE
 3000 CONTINUE
 3003 KZ=0
      DO 3500 I=1,NRLP
      NT=NIK(I)
 3550 NB=0
      DO 3600 J=1,NT
      JP=JNL(I,J)
      KIP=IABS(LP(I,J))
      IF (HD(JP)) 3610,3612,3610
```

```
 3612 NB=NB+1
      GO TO 3600
 3610 IF (J-1) 3600,3600,3615
 3615 JP1=JNL(I,J-1)
      IF (HD(JP1)) 3600,4605,3600
 4605 DO 4700 K=1,NPUMP
      IF (KIP-KP(K)) 4700,4610,4700
 4610 HP1=A(K)*(ABS(Q(KIP,KB)))**2.+B(K)*
     *ABS(Q(KIP,KB))+HO(K)
      GO TO 3625
 4700 CONTINUE
      HP1=0.
 3625 IF (LP(I,J)) 3630,3635,3635
 3630 HD(JP1)=HD(JP)-XKT(KIP)+HP1
      GO TO 3637
 3635 HD(JP1)=HD(JP)+XKT(KIP)+HP1
 3637 NB=NB-1
 3600 CONTINUE
      IF (NB-NT+1) 3501,3505,3505
 3501 IF (NB) 3500,3500,3550
 3505 KZ=1
 3500 CONTINUE
      IF (KZ) 3901,3901,3003
 3901 DO 4000 I=1,NPRV
      NPV=NVALV(I)
      KPV=NBJ(I)
      IF (Q(NPV,KB)) 4010,4010,4015
 4010 HGLU(I)=HD(KPV)
      GO TO 4000
 4015 HGLU(I)=HD(KPV)-XKT(NPV)*XLPRV(I)/(XL(NPV)-XLPRV(I))
      HGL(I)=VALST(I)
 4000 CONTINUE
      PRINT 4350
 4350 FORMAT (///16X,'H Y D R A U L I C  G R A D E S  O F  N O D E S')
      PRINT 4360
 4360 FORMAT (16X,'**********************************************'/)
      DO 4500 J=1,NJ
      PRINT 4400, J, HD(J)
 4400 FORMAT (16X,'HYDRAULIC GRADE OF NODE NO. ',I2,' = ',F8.2,
     *'  FT'/)
 4500 CONTINUE
      PRINT 4510
 4510 FORMAT (///16X,'U P S T R E A M / D O W N S T R E A M')
      PRINT 4515
 4515 FORMAT (16X,'G R A D E S  O F  P R V S')
      PRINT 4520
 4520 FORMAT (16X,'**************************************')
      DO 4450 I=1,NPRV
      PRINT 4455
 4455 FORMAT (/16X,'HYDRAULIC GRADE IMMEDIATELY')
      PRINT 4456, I, HGLU(I)
 4456 FORMAT (16X,'UPSTREAM OF PRV NO. ',I4,' = ',F8.2,
     *'  FT'/)
      PRINT 4460
 4460 FORMAT (16X,'HYDRAULIC GRADE IMMEDIATELY')
      PRINT 4461, I, HGL(I)
 4461 FORMAT (16X,'DOWNSTREAM OF PRV NO. ',I2,' = ',F8.2,
     *'  FT'/)
 4450 CONTINUE
      KB=KB-1
      PRINT 930,KB
  930 FORMAT (///16X,'NO. OF ITERATIONS = ',I2/)
      PRINT 990, ERR
  990 FORMAT (16X,'RELATIVE ERROR= ',F10.6)
      PRINT 994
  994 FORMAT ('1')
      STOP
      END

      SUBROUTINE GAUSS(A,X,N)
      DIMENSION A(110,111), X(100), Y(100)
      M=N+1
      N2=N-1
      DO 800 II=1,N2
      III=II+1
      DO 20 I=II,N
   20 Y(I)=ABS(A(I,II))
      KK=0
      TR=Y(II)
      DO 11 I=III,N
      IF (Y(I)-TR) 11,11,12
   12 TR=Y(I)
      KK=I
   11 CONTINUE
```

```
      IF (KK) 13,14,13
   13 DO 15 I=II,M
      STORE=A(KK,I)
      A(KK,I)=A(II,I)
   15 A(II,I)=STORE
   14 K=II+1
      DO 800 I=K,N
      DO 800 J=K,M
  800 A(I,J)=A(I,J)-A(I,K-1)*A(K-1,J)/A(K-1,K-1)
      X(N)=A(N,M)/A(N,N)
      DO 86 K=2,N
      J=M-K
      L=J+1
      X(J)=0.
      DO 87 I=L,N
   87 X(J)=X(J)+A(J,I)*X(I)
   86 X(J)=(A(J,M)-X(J))/A(J,J)
      RETURN
      END

$ENTRY
```

P I P E   D I S C H A R G E
****

DISCHARGE IN PIPE NO. 1 = 2.53 CFS

DISCHARGE IN PIPE NO. 2 = -0.38 CFS

DISCHARGE IN PIPE NO. 3 = 2.47 CFS

DISCHARGE IN PIPE NO. 4 = 0.72 CFS

DISCHARGE IN PIPE NO. 5 = 0.92 CFS

DISCHARGE IN PIPE NO. 6 = 1.08 CFS

DISCHARGE IN PIPE NO. 7 = 1.81 CFS

DISCHARGE IN PIPE NO. 8 = 3.19 CFS

H E A D   L O S S E S
****

HEAD LOSS IN PIPE NO. 1 = 128.15 FT

HEAD LOSS IN PIPE NO. 2 = 2.64 FT

HEAD LOSS IN PIPE NO. 3 = 122.03 FT

HEAD LOSS IN PIPE NO. 4 = 8.50 FT

HEAD LOSS IN PIPE NO. 5 = 19.90 FT

HEAD LOSS IN PIPE NO. 6 = 22.65 FT

HEAD LOSS IN PIPE NO. 7 = 6.23 FT

HEAD LOSS IN PIPE NO. 8 = 17.73 FT

H Y D R A U L I C   G R A D E S   O F   N O D E S
****

HYDRAULIC GRADE OF NODE NO. 1 = 173.77 FT

HYDRAULIC GRADE OF NODE NO. 2 = 57.58 FT

HYDRAULIC GRADE OF NODE NO. 3 = 60.22 FT

HYDRAULIC GRADE OF NODE NO. 4 = 182.27 FT

HYDRAULIC GRADE OF NODE NO. 5 = 37.56 FT

U P S T R E A M / D O W N S T R E A M
G R A D E S   O F   P R V S
****

HYDRAULIC GRADE IMMEDIATELY
UPSTREAM OF PRV NO. 1 = 50.12 FT

HYDRAULIC GRADE IMMEDIATELY
DOWNSTREAM OF PRV NO. 1 = 50.00 FT

NO. OF ITERATIONS = 6

RELATIVE ERROR= -0.000268