# WATER RESOURCES research center

Publication No. 105

#### EFFICIENCY/EQUITY ANALYSIS OF WATER RESOURCE PROBLEMS

bу

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# UNIVERSITY OF FLORIDA

#### Publication No. 105

#### of the

#### FLORIDA WATER RESOURCES RESEARCH CENTER

#### EFFICIENCY/EQUITY ANALYSIS OF WATER RESOURCE PROBLEMS

#### by

Stephen N. Payne

A thesis presented to the Graduate School of the University of Florida in partial fulfillment of the requirements for the degree of Master of Engineering

University of Florida

#### ACKNOWLEDGMENTS

I sincerely want to thank my graduate advisor and supervisory committee chairman, Dr. James P. Heaney, for his encouragement, guidance, and patience during all phases of my work while here at the University of Florida. Particularly, the time he spent guiding and reviewing this thesis research is immensely appreciated. I also wish to thank my other supervisory committee members, Dr. Paul A. Chadik, and Dr. Sanford V. Berg. Additionally, I'm truly grateful to the U.S. Air Force for making my pursuit of this Master of Engineering degree possible.

Most of all I want to express my deepest love and appreciation to my wife, Debbie, for her love, patience, and understanding during my struggles to complete this work.

# TABLE OF CONTENTS

	page
ACKNOWLEDGMENTS	ii
LIST OF TABLES	. <b>v</b>
LIST OF FIGURES	. vi
ABSTRACT	vii
CHAPTERS	
1 INTRODUCTION	. 1
2 ALLOCATING TREATMENT COST AMONG MULTIPLE POLLUTANTS	. 3
Introduction Problem Description Cost Allocation and Game Theory Physical Measure Method of Cost Allocation Game Theoretic Solution Notions Game Theoretic Cost Allocation Shapley Value Cost Allocation Simplified Shapley Value Cost Allocation Summary and Conclusions	3 9 13 18 19 21 25 28
<ul> <li>3 EFFICIENCY/EQUITY ANALYSIS.</li> <li>Introduction</li> <li>Efficiency Analysis</li> <li>Equity Analysis</li> <li>Two-part Pricing</li> <li>"Go-alone" Users</li> <li>Multiple Member Coalitions</li> <li>Parallel networks</li> <li>Series networks</li> <li>Modeling Network</li> <li>Net Benefits and Costs of the Various Coalitions</li> <li>"Go-alone" Net Benefits and Costs</li> <li>Two-member Coalitions</li> </ul>	30 31 32 33 33 37 39 40 42 45 45 45 53 58

	Further Discussion Summary and Conclusions	60 63				
4	CONCLUSIONS	65				
APPENDICES						
Α	COST FUNCTION PARAMETER DOCUMENTATION	69				
В	TREATMENT SYSTEM REMOVAL EFFICIENCIES AND COST PARAMETERS	84				
С	MARGINAL COSTS OF POLLUTANT REMOVAL	92				
D	SOLUTIONS TO NET BENEFITS EQUATIONS	95				
REFERENCES						
BIOGRAPHICAL SKETCH						

-

# LIST OF TABLES

Table		page
2-1	Pollutant influent and effluent concentrations with associated required removal efficiencies	5
2-2	Descriptions of the twelve most commonly used treatment systems	6
2-3	Technical removal efficiencies for treatment systems listed in Table 2-2	8
2-4	Cost parameters, a and b, for the assumed form of the annualized cost function, $C=a^*Q^b * 10^{6}$ , for each of the twelve treatment systems for three ranges of flow. Cost are in 1986 dollars.	10
2-5	Allocation of the total annual treatment cost using system 11 based on quantity removal	14
2-6	The subset of treatment systems meeting coalition c(1), BOD <sub>5</sub> removal, criteria	. 16
2-7	Characteristic function costs for the 15 possible coalitions	. 17
2-8	Coalition formation sequences and the associated marginal costs for each player	22
3-1	"Go-alone" user expected total revenues generated from marginal cost pricing with corresponding network costs and necessary annual charging system costs	54
3-2	Table of optimal coalition configurations based on requiring           efficient         per-unit charges for members of a given coalition	56
3-3	Table of optimal coalition configurations based on requiringuniform per-unit charges for members of a given coalition	62

V

# LIST OF FIGURES

Figure		<u>page</u>
2-1	Treatment system 1, primary treatment, typical flow diagram	. 7
3-1	Conditions for economic efficiency with economies of scale. (a) Maximum net benefit occurs at Q* where slope of TC equals slope of B. The fixed charge is y-intercept of tangent line, YZ, with TC at point Z with slope of TC curve at Z. (b) Variable charge for Q* is equal to marginal cost for Q*	36
3-2	Multi-member network formation arrangements	38
3-3	Network model for water supply system example	43
3-4	Demand functions for the users of the modeled water supply network	44
3-5	User 1 "go-alone" (a) benefits versus costs versus net benefits and (b) demand versus marginal costs	47
3-6	User 2 "go-alone" (a) benefits versus costs versus net benefits and (b) demand versus marginal costs	48
3-7	User 3 "go-alone" (a) benefits versus costs versus net benefits and (b) demand versus marginal costs	49
3-8	User 1 total revenue versus total cost	50
3-9	User 2 total revenue versus total cost	51
3-10	User 3 total revenue versus total cost	52

Abstract of Thesis Presented to the Graduate School of the University of Florida in Partial Fulfillment of the Requirements for the Degree of Master of Engineering

#### EFFICIENCY/EQUITY ANALYSIS OF WATER RESOURCE PROBLEMS

By

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April 1988

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Efficiency and equity analysis should be considered during the planning stage for water resource systems involving multiple products and/or participants. An efficiency analysis is conducted to find the most economically optimal system that maximizes benefits minus costs. An equity analysis is done to apportion the cost of the efficient system. This thesis addresses the relationship between these two issues.

Two general problems with applying these concepts to water resource systems are investigated. The first is that of equity as it concerns the allocation of treatment costs among multiple pollutants in a treatment facility. In this case, the most efficient treatment method is considered to be that treatment scheme that provides the required level of treatment at the minimum cost. Secondly, the effects of project economies of scale on efficiency and equity analysis are explored. Estimating the cost of removal for a particular pollutant in a treatment plant in which more than one type pollutant is removed can be a problem. The cost allocation dilemma arises for example in an activated sludge unit that removes 5-day biochemical oxygen demand (BOD5), has luxury uptake of phosphorus, and removes suspended solids in the secondary clarifier. The question becomes "Should BOD5 be charged the whole cost of the activated sludge unit?" If not, then by what method should the cost be apportioned among the various pollutants removed? Allocating costs based on a physical measure is compared to and shown to be inferior to methods based on the game theoretic Shapley value and simplified Shapley value. For this comparison, an example of allocating the sewage treatment cost of a 20 million gallon per day average daily flow plant designed to remove BOD5, total suspended solids, total phosphorus, and nitrogen is presented.

Secondly, cost economies of scale are investigated to see how they affect efficiency and equity analysis of multiple member water supply networks. Where economies of scale exist, it is shown that a sustainable two-part pricing scheme can be devised for which the variable per-unit and fixed fee portions of such a scheme are easily determined. A general relationship of the efficient variable charge to marginal cost is developed for single and multi-member networks involving both series and parallel network configurations. A modification to the remaining benefits cost allocation method is used to calculate a sustainable allocation of the costs for an example three-member water resource supply network.

viii

#### CHAPTER 1 INTRODUCTION

Numerous techniques are used to apportion costs of a water resource project among the participants or purposes. For any technique to be defensible it should meet both efficiency and equity criteria. An efficient system maximizes the net benefits of participants. However, concurrent with determining the most efficient system, the cost allocation method necessary to support this system should be developed. Equity analysis deals with making this allocation so that the apportionment of cost is acceptable to all participants. A game theoretic approach is used to help develop these ideas.

The first issue is that of equity as it concerns the allocation of treatment costs among multiple pollutants in a treatment facility. Chapter 2 deals with the problem of estimating the cost of removal for a particular pollutant in a treatment plant in which more than one type pollutant is removed. The cost allocation dilemma arises for example in an activated sludge unit that removes 5-day biochemical oxygen demand (BOD5), has luxury uptake of phosphorus, and removes suspended solids in the secondary clarifier. The question becomes "Should BOD5 be charged the whole cost of the activated sludge unit?" If not, then by what method should the cost be allocated among the various pollutants removed? Allocating costs based on a physical measure is compared to allocating the costs based on the game theoretic Shapley value and simplified Shapley value. For this comparison, an example is presented for allocating the sewage treatment cost of a 20 million gallon per day average daily flow plant

designed to remove BOD5, total suspended solids, total phosphorus, and nitrogen.

Secondly, cost economies of scale are investigated to see how they affect efficiency and equity analysis. Chapter 3 presents the economic efficiency and equity issues by looking at an example of a three-member water resources supply network. The specific task conducted is the determination of a sustainable cost allocation for the network participants which supports the economically efficient network. The allocation scheme proposed is a modification to the remaining benefits method.

Conclusions and recommendations are presented in Chapter 4.

#### CHAPTER 2 ALLOCATING TREATMENT COST AMONG MULTIPLE POLLUTANTS

#### Introduction

This chapter presents the game theoretic approach of using a simplified Shapley value as a solution to the sewage treatment plant (STP) cost allocation problem involving multiple pollutants. The type and degree of treatment necessary is a direct function of the regulatory effluent standards. The STP products are waters meeting the prescribed reductions in concentrations of specified pollutants. For example, if the only standard is that a 5-day biochemical oxygen demand ( $BOD_5$ ) discharge shall be less than 10 milligram per liter (mg/L) then the one relevant STP product is treated water with  $BOD_5$  less than 10 mg/L. However, multiple products may require consideration if an effluent standard is specified, not only for BOD<sub>5</sub> but also for total suspended solids (TSS) to be less than 5 mg/L, total phosphorus (Tot-P) to be less than 0.5 mg/L, and nitrogen (NH<sub>3</sub>-N) to be less than 1.9 mg/L. In this latter case, the STP has four relevant products, BOD<sub>5</sub> removed to meet the 10 mg/L BOD<sub>5</sub> standard, TSS removed to meet the 5 mg/L TSS standard, Tot-P removed to meet the 0.5 mg/L Tot-P standard, and NH<sub>3</sub>-N removed to meet the 1.9 mg/L standard. The challenge is to fairly allocate the total STP cost among the various pollutants being removed.

Cost allocations based on the physical measure of pounds of pollutant removed per day are compared to those based on the game theoretic

Shapley value and simplified Shapley value. The intent is to show that the game theoretic approach provides a fairer allocation of the overall treatment cost to the various pollutants and that a simplification of the Shapley value calculation makes it an attractive cost allocation scheme for sewage treatment system cost. Annualized cost will be used throughout.

#### **Problem Description**

Consider the following scenario. A STP is required to treat a 20 million gallon per day (MGD) average flow having the influent pollutant concentrations and required effluent concentrations listed in Table 2-1. The objective is to fairly allocate the treatment cost among the four pollutants. To develop the cost of treatment, the type of treatment system that will provide the required removal efficiencies must first be selected. The system of choice is the treatment system able to obtain the required removal efficiencies at a minimum cost.

Most sewage treatment plants are multipurpose, that is, they typically remove more than one type pollutant. Table 2-2 presents a general description of the twelve most commonly used treatment systems (U.S. Environmental Protection Agency, 1976a). A flow diagram for the primary treatment system number 1 as adapted from the referenced document is presented in Figure 2-1 (U.S. Environmental Protection Agency, 1976a).

Table 2-3 presents a characterization of the technical removal efficiencies reported for the twelve systems. By inspection of Table 2-3, only systems 11 and 12 would meet the specified treatment conditions of Table 2-1.

Table 2-1 Polluta require	ant influent and effluent concen ed removal efficiencies.	trations with associated

Pollutant	Influent concentration, mg/L	Allowable effluent concentration, mg/L	Required efficiency	
BOD <sub>5</sub>	210	10	0.95	-
TSS	230	5	0.98	
Tot-P	11	0.5	0.95	
NH <sub>3</sub> -N	20	1.9	0.91	

Table 2-2.	Descriptions	of the twelve	most	commonly	used <sup>·</sup>	treatment
	systems.					

Treatment System #	System description
1	Primary treatment
2	Primary treatment w/metal-salt (FeCl <sub>3</sub> ) addition
3	Trickling filter
4	Trickling filter w/metal-salt (FeCl <sub>3</sub> ) addition
5	Activated sludge
6	Activated sludge w/metal-salt (Alum) addition
7	Activated sludge/Nitrification (Single stage)
8	Activated sludge/Nitrification/Denitrification
	(Three stages)
9	Activated sludge/Nitrification-Filtration (w/Alum)
10	Physical/Chemical
11	Activated sludge/Nitrification/Denitrification/
	w/Alum(Three stages)
12	Activated sludge/Nitrification/Denitrification/
	Activated carbon (w/Alum)





Treatment	ntPollutant removal efficiencyPollutant				
System #	BOD <sub>5</sub>	TSS	Tot-P	NH <sub>3</sub> -N	
1	0.38	0.57	0.18	0.00	
2	0.52	0.78	0.82	0.00	
3	0.79	0.74	0.27	0.10	
4	0.88	0.87	0.82	0.10	
5	0.90	0.91	0.36	0.15	
6	0.93	0.93	0.82	0.15	
7	0.95	0.91	0.27	0.90	
8	0.95	0.91	0.27	0.95	-
9	0.95	0.96	0.91	0.90	
10 <sup>-</sup>	0.98	0.98	0.91	. 0.00	
11	0.98	0.98	0.95	0.95	
12	0.99	0.98	0.95	0.95	

-

•

Table 2-3.	Technical removal efficiencies for Table 2-2.	treatment systems listed in

The treatment system of choice would be the least costly of these two systems.

The characteristic cost function parameters for each of the treatment systems are presented in Table 2-4 and were developed from data presented in Appendix H of the U.S. Environmental Protection Agency Areawide Assessment Manual, Volume III, [1976a] (Heaney, 1979). Development of the specific cost function parameters for each of the systems is documented in Appendix A. Cost data are updated using the Engineering News Record 1986 index factor (Engineering News Record, 1987).

#### Cost Allocation and Game Theory

A number of methods have been proposed for allocating water project cost (U.S. Environmental Protection Agency, 1988). Apportionment of the treatment cost among the pollutants treated in a given STP is necessary to establish a fair basis for charging-out its operational cost to its various influent contributors. A realistic concern is for the allocation method to be auditable. Accountants can audit an operational STP using a physical measure of quantity removed. One problem with using a game theoretic approach is that it does not allow for direct auditing of an existing facility in a real number sense. Cost data to develop game theoretic based allocations are hypothetical except for the "as-built" plant annual cost data. Development of the game theoretic cost data relies on a model to determine alternative method treatment costs which are then used to develop the allocations. Maintaining a reproducible, current, and consistent cost model can be an

Table 2-4. Cost parameters, a and b, for the assumed form of the annualized cost function,  $C = a^*Q^b * 10^6$ , for each of the twelve treatment systems for three ranges of flow. Cost are in 1986 dollars.

	Q, MGD						
System	0.1	- 1.0	1.0 -	10.0	10.0 -	100.0	
number	а	b	а	b	а	b	
1	0.2423	0.3247	0.2423	0.5557	0.1742	0.699	
2	0.3152	0.3895	0.3152	0.6193	0.2151	0.7852	
3	0.3535	0.3907	0.3535	0.5678	0.2206	0.7726	
4	0.4437	0.4303	0.4437	0.6368	0.2714	0.8502	
5	0.4075	0.4193	0.4075	0.5769	0.4148	0.5692	
6	0.4856	0.4849	0.4856	0.5836	0.3071	0.7826	
7	0.4301	0.446	0.4301	0.5921	0.3103	0.7339	
8	0.6099	0.4008	0.6099	0.6085	0.3956	0.7964	
9	0.5037	0.4943	0.5037	0.6639	0.4115	0.7517	
10	0.9637	0.5604	0.9637	0.6715	0.7409	0.7857	
11	0.8403	0.5076	0.8403	0.58	0.4817	0.8216	
12	0.9512	0.4992	0.9512	0.6707	0.694	0.8076	

accounting problem. However, such models are manageable and if maintained can allow for a fairer allocation of cost.

For decision purposes, an engineer often needs to know the associated cost of removing each pollutant in a multiple product system. Within a manufacturing perspective, for a single unit process with multiple products, the cost allocation problem is normally handled in one of two ways (Kaplan, 1982). One way is *by-product* costing which considers one or more of the products as the main product(s) and all other products as by-products. The majority of the cost is allocated to the main product while only charging byproducts with their specific costs. The second method is *joint-product* costing which considers all the products as joint products of the process. Joint products are considered to have the same value. The costs are allocated among the joint products in one of two ways: a physical measure method such as pounds of product produced, or a value method based on the monetary return of each product. To handle the STP cost allocation problem involving multiple products from a single treatment process, a joint-product physical measure of pounds of pollutant removed per day method as recommended in the U.S. Environmental Protection Agency Federal Guidelines, Industrial Cost Recovery Systems, MCD-45, [1976b] will be considered.

For any cost allocation to be acceptable, it should satisfy several properties; group rationality, individual rationality, and subgroup rationality. The total project cost must be covered by the participants or players in the project. This is called <u>group rationality</u> and is synonymous with Pareto optimality (Luce and Raiffa, 1967). Group rationality is one of three so called axioms of fairness (Shapley, 1971). Group rationality simply says the total

cost of the grand coalition, c(N), must be apportioned among the N players. In mathematical terms,

$$\sum_{i \in N} x(i) = c(N)$$
(2-1)

where x(i) is the allocated cost or the charge to player i. <u>Individual rationality</u> is another of the axioms and says that player i should not pay more than his go-it-alone cost, i.e.,

$$x(i) \le c(i), \quad \forall i \in \mathbb{N}.$$
 (2-2)

In game theoretic terms, the set of solutions or charges satisfying equations 2-1 and 2-2 is called the set of imputations. The third axiom is called <u>subgroup rationality</u> which is an extension of individual rationality to include subgroups. Simply stated, no subgroup or subcoalition S should be apportioned a cost greater than its go-it-alone cost, i.e.,

$$\sum_{i \in S} x(i) \le c(S), \quad \forall S \in N.$$
(2-3)

Subgroup rationality, in game theory terms, defines the core of the game given that the first two axioms are satisfied.

Another condition related to group rationality is <u>subadditivity</u>. Subadditivity is defined as

 $c(S) + c(T) \ge c(S \cup T)$   $S \cap T = \emptyset$ , and  $S,T \in N$  (2-4)

where;  $\emptyset = \text{empty set}$ 

S and T = any two disjoint subsets of N

N =  $\{1, 2, \dots, n\}$  represents the set of players in a game

S = any subset of players in N

c(S) = cost function assigning a real number cost to each nonempty subset of S players.

The least cost (optimal) solution for an S-member coalition, c(S), assumes that the (N-S)-member (complementary) coalition is not present (Ng, 1985). With c(S) defined as the least cost solution for coalition S for the situation where the (N-S)-member coalition is not present, the cost game is naturally subadditive. Subadditivity is a natural consequence for subsets S and T because the worst they can do as a coalition is the cost of independent action, or

 $c(S) + c(T) = c(S \cup T)$   $S \cap T = \emptyset$ , and  $S,T \in N$ . (2-5)

Subadditivity is a desirable characteristic for any coalition to ensure no members of the coalition will want to leave. In other words, it is a necessary condition for stability. Subadditivity is a more general condition which allows for both increasing marginal cost and increasing average cost over some range of outputs.

#### Physical Measure Method of Cost Allocation

Returning to the cost allocation problem, on a quantity removal basis, the allocation of the annual treatment cost for the optimal system 11 is shown in Table 2-5. By definition, the allocation in Table 2-5 satisfies group rationality and subadditivity conditions. But are individual and subgroup rationality satisfied? Other cost data are necessary to check these conditions. The cost of go-it-alone treatment for each of the pollutants is required as is the cost of treatment for all the various groupings or coalitions that could occur. To develop these costs, the coalitions and their associated costs must be

base	ed on quantity rem			
Pollutant	Removal required, mg/L	% of total	Allocated cost	"Go-alone" cost
BOD <sub>5</sub>	200.0	44.1	\$2,492,000	\$2,800,000

49.6

2.3

4.0

\$2,802,000

\$130,000

\$226,000

\_\_\_\_\_

\$5,650,000

Table 2-5.	Allocation of the total annual treatment cost for using system 11	
	based on quantity removal.	

5.	Allocation of the total annual treatment cost for usin based on quantity removal.

TSS

Tot-P

NH<sub>3</sub>-N

totals

225.0

10.5

18.1

=====

453.6

\$5,650,000 \$5,650,000 \$4,300,000

determined. For n players, there are  $2^{n}$ -1 possible coalitions excluding the empty set. In this example n = 4, therefore, 15 coalitions must be considered. The characteristic function of a coalition is the minimum cost treatment system among the twelve presented in Table 2-2 able to obtain the required removal efficiencies. For example, BOD<sub>5</sub> requires a removal efficiency  $\geq 0.95$  (see Table 2-1). From Table 2-3, the subset of systems consisting of numbers 7,8,9,10,11, and 12 are all able to meet this required removal efficiency. Table 2-6 presents the costs associated with this subset of systems. The supporting documentation for these cost determinations is presented in Appendix B.

By comparison of the annual cost in Table 2-6, system number 7 is the minimum cost system for  $BOD_5$  removal. Therefore, the assigned characteristic function, c(1), is that cost associated with system number 7 or \$2,800,000. A summary of the characteristic function cost for the 15 possible coalitions is presented in Table 2-7.

Comparison of the allocation of cost based on quantity of pollutant removed presented in Table 2-5 to the various coalition costs presented in Table 2-7, indicates that individual and subgroup rationality are satisfied. However, the vast difference between Tot-P and NH<sub>3</sub>-N go-it-alone treatment costs and their respective allocated costs under the quantity method suggests a more equitable allocation should be made to better reflect the true marginal cost of their removal. Clearly if TSS is not treated then cost allocation based on a quantity measure fails the individual rationality for BOD<sub>5</sub>; i.e., \$5,560,000 \* 87.5% = \$4,944,000 which is greater than c(1) of \$2,800,000. This indicates the possible problem with using a physical measure of use for allocating cost.

Table 2-6.	The subset of	treatment	systems	meeting	coalition	c(1),
	BOD <sub>5</sub> removal,	criteria.				

Sys.	R	Removal	Efficienci	es		Annual cos	st parameters
No.	BOD <sub>5</sub>	TSS	Tot-P	NH <sub>3</sub> -N	а	b	aQ^b
7	0.95	0.91	0.27	0.90	0.3103	0.7339	\$2,800,000
8	0.95	0.91	0.27	0.95	0.3956	0.7964	\$4,300,000
9	0.95	0.96	0.91	0.90	0.4115	0.7517	\$3,910,000
10	0.98	0.98	0.91	0.00	0.7409	0.7857	\$7,800,000
11	0.98	0.98	0.95	0.95	0.4817	0.8216	\$5,650,000
12	0.99	0.98	0.95	0.95	0.694	.0.8076	\$7,800,000

Note: Cost based on Q = 20 mgd.

	Minimum				
Coalition	Feasible systems	cost system	Cost(\$/yr)		
c(1) BOD <sub>5</sub> only	7,8,9,10,11,12	7	\$2,800,000		
c(2) TSS only	10,11,12	11	\$5,650,000		
c(3) Tot-P only	11,12	11	\$5,650,000		
c(4) NH <sub>3</sub> -N only	8,11,12	8	\$4,300,000		
c(12)	10,11,12	11	\$5,650,000		
c(13)	11,12	11	\$5,650,000		
c(14)	8,11,12	8	\$4,300,000		
c(23)	11,12	11	\$5,650,000		
c(24)	11,12	11	\$5,650,000		
c(34)	11,12	. 11	\$5,650,000		
c(123)	11,12	11	\$5,650,000		
c(124)	11,12	11	\$5,650,000		
c(134)	11,12	11	\$5,650,000		
c(234)	11,12	11	\$5,650,000		
c(1234)	11,12	11	\$5,650,000		

Table 2-7. Characteristic function costs for the 15 possible coalitions.

Cost allocation based on a game theoretic approach is suggested as an improvement to the physical measure.

#### Game Theoretic Solution Notions

Many game theoretic solution concepts have been developed over the years. One that has found favor in the cost allocation literature is the Shapley value, originally developed by Shapley in 1953. Before presenting the Shapley value solution to the STP cost allocation problem, the general concepts of game theory concerning cost allocation applications are introduced.

Of main interest are those games wherein all the players voluntarily cooperate in forming coalitions. A coalition is an agreement among players to coordinate their available strategies in such a way that all members of the coalition benefit (Levin and DesJardins, 1970). Such games are called cooperative games. The attractiveness of the coalition depends on the payoffs received by the players, where a payoff is the net benefit a player obtains by participating. A coalition is stable when no player or subset of players can improve on their payoffs by withdrawing from the game. All such stable coalitions are said to be in the core. Not all games necessarily have a core, but for those that do, the most desirable coalition will be in the core (Berg and Tschirhart, 1987).

Games with more than two players are called n-person games. The addition of players quickly complicates the analysis of coalition formation. If a game has n players, there are 2<sup>n</sup> possible coalitions including the empty set.

Cooperative games are normally studied in three forms (levels of abstraction): extensive form, normal form, or characteristic function form (Ng, 1985). The extensive form involves a complete description of the rules of a game and is generally characterized by a game tree to describe every player's move. The normal form condenses the description of a game into sets of strategies for each player and is represented by a game matrix. The characteristic function form describes a game in terms of payoffs rather than rules or strategies. Most efforts in cooperative game theory have been with games in the characteristic function form.

In addition to the form of the cooperative game, cooperative games can be of three types depending on whether the game is defined in terms of costs, savings, or values. Value games are games in which the players in the coalition seek to maximize profits or net revenues. Games in which the players seek to minimize costs are called cost games. Cost games can be converted to saving games by measuring savings relative to the cost of not participating in a coalition (Heaney and Dickinson, 1982). All further discussions in this chapter are about cooperative cost games.

#### Game Theoretic Cost Allocation

Cost allocation may involve conflicts of interest between participants in a project. Participants may be persons or groups and/or purposes who will pay a share of the cost of the project (Heaney and Dickinson, 1982). As stated earlier, the objective is to find an allocation of costs among these groups or purposes that fairly assigns a share of the cost to each and that assigns the

entire cost. Concepts from cooperative game theory can be used to find such a "fair" solution that is also efficient (Luce and Raiffa, 1958).

The cost allocation properties of subadditivity and fairness discussed earlier must also hold for any game theoretic approach. Concerning the core, in 1971, Shapley demonstrated that a core always exists for convex games (Ng, 1985). Convex games occur when the incentives for cooperation are relatively strong. Convex games are a special class of cooperative games. Convexity in a cost game is defined as

 $c(S) + c(T) \ge c(S \cup T) + (S \cap T)$   $S \cap T \ne \emptyset$ ,  $\forall S, T \in N$  (2-6)

or, written another way

$$c(S \cup i) - c(S) \ge c(T \cup i) - c(T) \qquad S \in T \in N - \{i\}, \ i \in N.$$
 (2-7)

In words, convexity means that the incremental cost for player i to join coalition T is less than or equal to the incremental cost for player i to join a subset of T (Ng, 1985). Convexity is analogous to economies of scale.

In the sewage treatment plant cost allocation problem, a special feature exists that allows a simplified Shapley value to be used. This feature is that the characteristic function is a cost function with the property that the cost of any subset of players is equal to the cost of the most expensive player in that subset (Littlechild and Owen, 1973). After developing the full Shapley value, the simplified Shapley value will be described.

#### Shapley Value Cost Allocation

The Shapley value, one of several game theoretic concepts, can be used to estimate each pollutant's associated cost of removal. The Shapley value for player i is defined as the incremental cost of player i entering the coalition averaged over all the possible orderings of player i's position when joining the coalition. That is to say, each player is expected to pay the incremental cost incurred by his addition to the coalition. Normally a player is better off to join a coalition as late as possible. Since the actual coalition formation sequence is assumed unknown, the Shapley value assigns an equal probability for all sequences of coalition formation, i.e., the probabilities of each player being the first to join are equal, as are the probabilities of joining second, third, etc. For a game having n players, n! formation sequences are possible and the probabilities of joining first or last is equal to 1/n while the probabilities of joining at some intermittent point is equal to  $1/[n^{*}(n-1)]$ . For a four-player game there are 4! or 24 possible sequences of formation. The formation sequences for the 4-player STP game and each player's associated marginal cost are presented in Table 2-8. Supporting documentation for the marginal cost determinations is included at Appendix C.

From the listing of possible formation sequences shown in Table 2-8, each player enters first or last n!/n or 6 times, which gives each player a 1/n =1/4 probability of being the first or last to enter the coalition. Additionally, each player has  $1/[n^*(n-1)] = 1/12$  probability of joining second or third behind any other one or two players.

The Shapley value has been criticized for falling outside the core for nonconvex games and for existing for games with no core (Hamlen et al., 1980). If convexity conditions are satisfied, then the Shapley value can safely

	Marginal cost (\$*10^6)				
	1	2	3	4	
Formation sequence	BOD <sub>5</sub>	TSS	Tot-P	NH <sub>3</sub> -N	
1234	2.80	2.85	0	0	
1243	2.80	2.85	0	0	
1324	2.80	0	2.85	0	
1342	2.80	0	2.85	0	
1423	2.80	1.35	0	1.50	
1432	2.80	0	1.35	1.50	
2134	0	5.65	0	0	
2143	0	<b>5</b> .65	0	0	
2314	0	5.65	0	0	
2341	0	5.65	0	0	
2413	0	5.65	0	0	
2431	0	5.65	0	0	
3124	0	0	5.65	0	
3142	0	0	5.65	0	
3214	0	0	5.65	0	
3241	0	0	5.65	0	
3412	0	0	5.65	0	
3421	0	0	5.65	0	
4123	0	1.35	0	4.30	
4132	0	0	1.35	4.30	
4213	0	1.35	0	4.30	
4231	0	1.35	0	4.30	
4312	0	0	1.35	4.30	
4321	0	0	1.35	4.30	
		====	=====	====	
mean	0.700	1.875	1.875	1.200	

Table 2-8.	Coalition formation	sequences	and the	associated	marginal
	costs for each playe	er.			-

-

be used to allocate cost. In fact, if a game is convex, the Shapley value is in the center of the core (Shapley, 1971). Because the Shapley value may be outside the core for a nonconvex game, the convexity of the STP cost allocation game must be verified. By observation of the marginal cost (MC) data presented in Table 2-8, the condition of convexity can be proven. Each player's MC is observed to decrease as it joins later in a particular formation sequence following a particular player. This decreasing MC property satisfies the condition of convexity.

Given the above explanation of formation sequences and marginal cost, the Shapley value or allocated cost for player 1 in the four-person game is

$$\Phi(1) = \frac{1}{4}c(1) + \frac{1}{12}[c(12) - c(2)] + \frac{1}{12}[c(13) - c(3)] \qquad (2-8)$$
  
+  $\frac{1}{12}[c(14) - c(4)] + \frac{1}{12}[c(123) - c(23)]$   
+  $\frac{1}{12}[c(124) - c(24)] + \frac{1}{12}[c(134) - c(34)]$   
+  $\frac{1}{4}[c(1234) - c(234)]$ 

The general formula used to calculate the Shapley value is

$$\bigoplus_{S=1}^{n} (i) = \sum_{S=1}^{n} \frac{(s-1)! (n-s)!}{n!} [c(S) - c(S-i)]$$
(2-9)

where i = player number (i = 1,2, \*\*\*, n)
 n = number of players
 S = subset coalition within n
 c(S) = cost of coalition subset S
 c(S-i) = cost of coalition subset S without player i
 s = number of players in coalition S
[c(S+i) - c(S)] = the incremental cost of adding player i to the S coalition.

Though normally considered theoretically sound, the Shapley value is often criticized for being computationally burdensome especially for games with a large number of players. For an n-player game, the Shapley value for each player requires the computation of  $2^{n-1}$  probability coefficients and incremental costs. The four-player STP game involves determining 8 coefficients and incremental costs to calculate the charge for each player. The probability coefficients are really no problem computationally since only two calculations are involved, 1/n for the probability of joining first or last and  $1/[n^*(n-1)]$  for the probability of joining other than first or last. However, making sure all the possible incremental costs have been included requires some close tracking.

Using the Shapley value general formula, the allocated cost to each of the four players in the STP game are (Reference the characteristic function costs for the possible coalitions presented in Table 2-7)

$$\begin{split} & (1) = \frac{1}{4}c(1) + \frac{1}{12}[c(12) - c(2)] \\ & + \frac{1}{12}[c(13) - c(3)] + \frac{1}{12}[c(14) - c(4)] \\ & + \frac{1}{12}[c(123) - c(23)] + \frac{1}{12}[c(124) - c(24)] \\ & + \frac{1}{12}[c(134) - c(34)] + \frac{1}{4}[c(1234) - c(234)] \\ \\ & (1) = \frac{1}{4}(2,800,000) + \frac{1}{12}(5,650,000 - 5,650,000) \\ & + \frac{1}{12}(5,650,000 - 5,650,000) + \frac{1}{12}(4,300,000 - 4,300,000) \\ & + \frac{1}{12}(5,650,000 - 5,650,000) + \frac{1}{12}(5,650,000 - 5,650,000) \\ & + \frac{1}{12}(5,650,000 - 5,650,000) + \frac{1}{14}(5,650,000 - 5,650,000) \\ & + \frac{1}{12}(5,650,000 - 5,650,000) + \frac{1}{4}(5,650,000 - 5,650,000) \\ & + \frac{1}{12}(5,650,000 - 5,650,000) + \frac{1}{4}(5,650,000 - 5,650,000) \\ & + \frac{1}{12}(5,650,000 - 5,650,000) + \frac{1}{4}(5,650,000 - 5,650,000) \\ & + \frac{1}{12}(5,650,000 - 5,650,000) + \frac{1}{4}(5,650,000 - 5,650,000) \\ & + \frac{1}{12}(5,650,000 - 5,650,000) + \frac{1}{4}(5,650,000 - 5,650,000) \\ & + \frac{1}{12}(5,650,000 - 5,650,000) + \frac{1}{4}(5,650,000 - 5,650,000) \\ & + \frac{1}{12}(5,650,000 - 5,650,000) + \frac{1}{4}(5,650,000 - 5,650,000) \\ & + \frac{1}{12}(5,650,000 - 5,650,000) + \frac{1}{4}(5,650,000 - 5,650,000) \\ & + \frac{1}{12}(5,650,000 - 5,650,000) + \frac{1}{4}(5,650,000 - 5,650,000) \\ & + \frac{1}{12}(5,650,000 - 5,650,000) + \frac{1}{4}(5,650,000 - 5,650,000) \\ & + \frac{1}{12}(5,650,000 - 5,650,000) + \frac{1}{4}(5,650,000 - 5,650,000) \\ & + \frac{1}{12}(5,650,000 - 5,650,000) \\ &$$

 $\Phi(1) = 1/4(2,800,000) =$ \$700,000/year.

Likewise,

 $\Phi(2) =$ \$1,875,000/year

 $\Phi(3) =$ \$1,875,000/year

 $\Phi(4) =$ \$1,200,000/year

and,

$$\phi(1) + \phi(2) + \phi(3) + \phi(4) =$$
\$5,650,000.

The sum of the Shapley values for the four players equals the grand coalition cost, c(1234), which satisfies group rationality and subadditivity. Additionally, by comparing these allocations to the costs presented in Table 2-7 for the various coalitions, individual and subgroup rationality are also found satisfied.

Intuitively, the Shapley value can be understood as a "marginal cost average." If the marginal cost for each player is determined for each possible formation sequence using total enumeration, then the Shapley value is simply the average of the marginal costs over all possible orders of formation. This is illustrated in Table 2-8 as the mean for each player's column of marginal costs.

For all of the marginal costs listed in Table 2-8, 60% are equal to zero and the rest only consist of six unique costs. If the marginal cost calculations leading to zero could be avoided, significantly fewer calculations would be necessary to develop the respective Shapley values. This observation led *Littlechild and Owen* [1973] to propose a simplification of the Shapley value method.

#### Simplified Shapley Value Cost Allocation

As indicated in the previous set of calculations, the determination of the Shapley values for the players in a four-person game gets lengthy. However, due to a special quality of the characteristic function of the various possible coalitions presented in Table 2-7, a simplification of the Shapley value determination is possible. The specific quality is that the largest cost of separate action by any of the players in a particular coalition dictates the characteristic function cost of the whole coalition. For example, any coalition containing player 2 will have a characteristic function cost equal to that of player 2 going alone. This is because for the four possible separate actions: c(1), c(2), c(3), and c(4), c(2) has a cost equal to the maximum of any of the four possible separate actions. Therefore, coalitions c(123), c(124), c(23), etc. all have the same cost of \$5,560,000 which is the cost of separate action for c(2).

*Littlechild and Owen* [1973] proved for this special case the general Shapley value equation could be represented by

$$\phi_{i} = \phi_{i-1} + (C_{i} - C_{i-1})/r_{i}$$
(2-10)

where:  $c_i = cost$  associated with player type i

- $c_{i-1} = cost$  associated with player type i-1 and  $c_0=0$ 
  - $\phi_i$  = Shapley value assigned cost to player type i
- $\phi_{i-1}$  = Shapley value assigned cost to player type i-1 and  $\phi_0$ =0
  - $r_i = (to determine r_i two new terms must be defined)$
  - m = number of player types; i.e., the number of unique separate (single player coalition) action costs ordered from least to most costly; and,
  - n<sub>i</sub> = number of players associated with player type i, where i = 1 to m

and finally,

$$r_i = \sum_{i=k}^{m} n_i$$
 for k = 1,2, •••, m. (2-11)

For the STP player coalitions shown in Table 2-7, the separate actions c(1), c(2), c(3), and c(4), only have three unique costs of \$2,800,000, \$5,650,000, and \$4,300,000. Ordering these unique costs from least to most

costly, the three unique player types are: Player type 1 being associated with \$2,800,000 and consisting of c(1) only meaning  $n_1 = 1$ ; player type 2 being associated with \$4,300,000 and consisting of c(4) only meaning  $n_2 = 1$ ; and player type 3 being associated with \$5,650,000 and consisting of c(2) and c(3) meaning  $n_3 = 2$ . The simplified Shapley calculations for this example are

$$r_{1} = \sum_{i=1}^{3} n_{i} = 1 + 1 + 2 = 4$$
  
$$r_{2} = \sum_{i=2}^{3} n_{i} = 1 + 2 = 3$$
  
$$r_{3} = \sum_{i=3}^{3} n_{i} = 2$$

and, the simplified Shapley value assigned cost to each of the player types are

$$\begin{split} \varphi_1 &= \varphi_0 + (c_1 - c_0)/r_1 = 0 + 2,800,000/4 = \$700,000 \\ \varphi_2 &= \varphi_1 + (c_2 - c_1)/r_2 = 700,000 + (4,300,000 - 2,800,000)/3 = \$1,200,000 \\ \varphi_3 &= \varphi_2 + (c_3 - c_2)/r_3 = 1,200,000 + (5,650,000 - 4,300,000)/2 = \$1,875,000. \\ \end{split}$$
From the assigned cost for each player type, the allocated cost to the four players whose types are  $\varphi(1)$  is type  $\varphi_1$ ,  $\varphi(2)$  is type  $\varphi_3$ ,  $\varphi(3)$  is type  $\varphi_3$ , and  $\varphi(4)$  is type  $\varphi_2$ , becomes

$$\Phi = \{$$
 \$700,000, \$1,875,000, \$1,875,000, \$1,200,000 $\}$ 

which is identical to the general Shapley value allocations calculated earlier.
The simplified Shapley method is much simpler than the general Shapley method. This advantage in computational ease makes the simplified Shapley method attractive. In general, the simplified Shapley method can be used when the facility being costed-out produces multiple products, the facility built to handle the maximum cost player will satisfy the requirements of the other players with the cost of any coalition of players being governed by the maximum cost for any single player in the coalition, and the game is convex so that the Shapley value is in the center of the core. These conditions were true in the STP cost game since the system selected for the grand coalition, System 11, not only provided the required removal efficiency for TSS and Tot-P but also met the requirements of BOD<sub>5</sub> and NH<sub>3</sub>-N. Additionally, System 8 for NH<sub>3</sub>-N being more costly than System 7 for BOD<sub>5</sub> provided the required removal efficiency for BOD<sub>5</sub> which meets the condition that a more costly system satisfy the requirements of any less costly system requirements.

# Summary and Conclusions

An engineer trying to estimate the cost of removing a particular pollutant in a treatment plant which removes more than one type of pollutant needs an acceptable cost allocation method. The cost allocation dilemma arises for example in an activated sludge unit that removes  $BOD_5$ , has luxury uptake of phosphorus, and removes suspended solids in the secondary clarifier. The question becomes "Should  $BOD_5$  be charged the whole cost of the activated sludge unit?" If not, then by what method should the cost be allocated among the various pollutants removed? This chapter presents a comparison of allocating costs based on a physical measure to the game theoretic Shapley value. A general introduction to basic game theory concepts and a cost allocation example are presented for a 20 MGD treatment plant designed to remove the four pollutants: 5-day biochemical oxygen demand (BOD<sub>5</sub>), total suspended solids (TSS), total phosphorus (Tot-P), and nitrogen (NH<sub>3</sub>-N). Due to a special property of the various treatment schemes, a simplification of the Shapley value calculation is demonstrated which greatly eases the more tedious calculations of the Shapley value for large games. Using the simplified Shapley value a fair set of pollutant charges can be estimated without relying directly on a physical measure of use, such as pounds removed per day. This same cost allocation procedure would work for other combinations of pollutants to be removed in water and wastewater treatment plants including handling of hazardous waste. Even though a game theoretic approach to cost allocation involves hypothetical cost that must be estimated using cost models, it is still a viable cost allocation method that should not be overlooked.

# CHAPTER 3 EFFICIENCY/EQUITY ANALYSIS

# Introduction

Many situations exist where groups can take advantage of economies of scale in production and distribution costs. One such situation can be a regional water resource system which can be an economically attractive alternative to separate systems. However, this regional system may not be simple to organize and manage. Many complex socio-political, legal and other problems must be resolved. This section only addresses the problem of determining the economically efficient system while devising a cost allocation scheme that allows the efficient system to be maintained.

Efficiency analysis determines the economically optimal system from the standpoint that such a system maximizes benefits minus costs. Once the most economically efficient system is determined, a method of allocating the system cost to the participants must be devised. Equity analysis involves determining an equitable allocation of these costs to convince each participant that the economically optimal regional system is their best alternative.

Efficiency and equity analysis have been considered as separate issues (Loughlin, 1977). However, it makes little sense to determine the most efficient system if no sustainable cost allocation can be devised. In other words, these two issues are inseparable since one directly affects the other. Rossman (1978) and Zajac (1978) support the idea that efficiency and equity analysis should occur concurrently. Proefke (1984) examined the relationship between efficiency and equity using a small wastewater reuse project. She

also argues that the two issues should be considered together. Ng (1985) investigated the efficiency/equity analysis issue and he too concluded they must be considered concurrently.

The following is an example regional water supply network efficiency/equity analysis using a total enumeration procedure and concepts from cooperative game theory. The specific task is to determine the regional water supply network which maximizes net benefits and then to determine a "sustainable" cost allocation where, "sustainable" means that the axioms of fairness discussed in the previous chapter of group rationality, individual rationality, and subgroup rationality are satisfied. Cost allocation using a modification to the remaining benefits method will be presented.

# Efficiency Analysis

Finding the economically efficient or optimal regional water supply system will be modeled as a mathematical optimization problem. The optimal system maximizes benefits minus costs. In economic terms the most efficient point occurs where marginal benefit equals marginal cost (Henderson and Quandt, 1980). Marginal costs represent supply while marginal benefits depict the demand curve. Demand curves define the relationship of price and quantity demanded. However, total system costs may not be covered if prices are set at marginal cost. This raises the issue of the need to deviate from marginal cost pricing which introduces inefficiencies into the system either through over or under consumption. The objective of the equity or acceptability analysis will be to find an allocation scheme that allows marginal cost per-unit pricing while recovering the remaining system costs through a fixed, nonmarginal charge.

### Equity Analysis

As stated earlier, regional systems should not be decided upon solely on the basis of economic efficiency but one must also devise an acceptable cost allocation. Equity analysis is the process of finding that acceptable allocation. Many pricing strategies have been proposed in the past to ensure total revenues cover total costs. James and Lee (1971) categorize these strategies into three basic types: price discrimination, marginal cost pricing plus an additional fixed fee, and marginal cost pricing plus subsidies. The allocation procedure presented is a modification of the remaining benefits methods which allows the net benefit of each member to be maintained or increased relative to their "go-alone" option while increasing the total net benefit to the group. The proposal is a two-part pricing scheme in which each participant pays a fixed fee plus a per-unit charge equal to marginal cost. The assumption is that the user will not adjust quantity demanded based on the fixed charge but only on the per-unit charge thus allowing efficient usage while still covering total system costs.

In evaluating the proposed remaining benefits cost allocation method with modifications, the principles of game theory relating to the core of individual, subgroup, and group rationality must be maintained. Thus, no individual member or subgroup of members will be assigned a net benefit less than their net benefit of nonparticipation, while assuring that total costs must be apportioned among the members. The net benefit of nonparticipation is that maximum net benefit that each member and each subgroup would obtain by acquiring their most economically efficient level of service independently. Therefore, as discussed in the game theoretic solutions notions section of the previous chapter, efficiency/equity analysis for the regional water supply

system involving n members using a total enumeration technique requires determining 2<sup>n</sup>-1 solutions excluding the empty set. In order to do the cost allocation, the costs of the optimally efficient systems for each individual and each subgroup of members must be determined as these costs are the basis for the fixed fee part of the proposed two-part price.

#### Two-part Pricing

### "Go-alone" Users

where

Where cost economies of scale exist, it will be shown a two-part pricing scheme can be devised for which the variable and fixed charge portions can be easily determined. Consider the total cost estimating model representing the total cost function as a power function which is a frequently used model for water resource projects. The standard cost estimating model becomes

$C = a^*Q^b$	(3-1)
C = total cost in \$,	
Q = size of facility	

a = intercept of cost equation, \$/Q, and

b = exponent of cost equation.

For cases where b in equation 3-1 is less than 1, then unit costs decrease as size increases and economies of scale are said to exist.

Showing how economies of scale lead to a natural two-part pricing scheme is easily done graphically. However, before showing the graphic, the underlining principles are presented. For a given demand function, the most efficient point where maximum net benefits are achieved occurs where marginal cost is equal to marginal benefit. This is also the same point where the slope of the total cost curve equals the slope of the total benefit curve. The mathematical proof of this goes as follows:

Let	$P = y - m^*Q$	(3-2)
where	P = price as a linear function of Q in \$/Q,	
	Q = demand, mgd,	
	y = intercept of price equation, \$/Q, and	
	m = slope of price equation	

then	$B = \int P dQ$ , or	(3-3)
	$B = y^*Q - (m^*Q^2)/2$	
where	B = benefits in total \$,	

.

using	$C = a^*Q^b$	
then	$MC = dC/dQ = a^*b^*Q^{(b-1)}$	(3-4)
where	MC = marginal cost, \$/Q,	

and	NB = B - C	(3-5)
where	NB = net benefits in total\$,	
or	NB = y*Q - (m*Q^2)/2 - a*Q^b.	(3-6)

Since the second derivative of NB with respect to Q, for all positive b less than one, is always negative, the NB function is strictly concave and therefore the maximum net benefit occurs where the first derivative of NB with respect to Q is equal to zero, or

 $dNB/dQ = y - m^{*}Q - a^{*}b^{*}Q^{*}(b-1) = 0, \quad (3-7)$ but  $P = y - m^{*}Q, \text{ and}$  $MC = a^{*}b^{*}Q^{*}(b-1),$ 

therefore, P = MC. (3-8) Which says that the maximum net benefit point occurs for the Q where the slope of the net benefit function equals the slope of the total cost function or where marginal benefit equals marginal cost.

Thus, to maintain efficiency, the variable per-unit charge (VC) is set equal to marginal cost where it equals marginal benefit which is the same as using the slope of the total cost curve when it equals the slope of the total benefit curve. However, to cover total project cost a fixed charge (FC) often must be assessed. The required size of this fixed charge can be determined as the y-intercept of a line tangent to the total cost curve having the same slope as the total cost curve at the maximum net benefit point. This concept is represented graphically in Figure 3-1. By letting the line tangent to the total cost (TC) curve, shown as YZ in Figure 3-1, be represented as y(x)=mx + B, solving for B analytically gives the fixed charge necessary to cover the total project cost while allowing marginal cost pricing, or



Figure 3-1. Conditions for economic efficiency with economies of scale.
(a) Maximum net benefit occurs at Q\* where slope of TC equals slope of B. The fixed charge is y-intercept of tangent line, YZ, with TC at point Z with slope of TC curve at Z. (b) Variable charge for Q\* is equal to marginal cost for Q\*.

Let	C' = total cost of network at opting	mal flow
	C' = FC + VC	(3-9)
	VC = MC' * Q'	(3-10)
where	MC' = marginal cost of network at	optimal flow
	Q' = optimal flow for maximum r	net benefits

therefore FC = C' - (MC' \* Q') (3-11)

By observation of Figure 3-1, one can see that as economies of scale decrease, i.e., b increases, the corresponding fixed charge would decrease using this scheme until the point where b=1 and all charges are necessarily covered through variable charges. For the case where economies of scale do not hold, this concept is of less value since the fixed charge becomes negative. However, since economies of scale often occur for water resource projects this observation at least allows a first cut at determining an efficient two-part price. For "go-alone" users this idea is easily understood since the structure of the costs and benefit functions can be clearly represented and all the terms in the net benefit equation are separable.

# Multiple Member Coalitions

For multi-member networks there are three general formation arrangements: parallel, series, or some combination of parallel and series. These various arrangements are depicted in Figure 3-2. Evaluation of maximizing the net benefits for each of these cases will now be presented.



Figure 3-2. Multi-member network formation arrangements.

Parallel networks.	(See Figure 3-2a)
Let	$P_i = Y_i - m_i * Q_i$
where	P <sub>i</sub> = demand function for user i.

Assume the total annualized cost function for constructing the pipelines are characterized by economies of scale, i.e., 0 < b < 1, and expressed as a linear function of distance and a nonlinear function of flow; or

$$C = a^*Q^{b*}L_{ij}$$
(3-12)  
where  $C = \text{total annualized pipeline cost, 1000$/year,}$ 

Q = quantity of flow, mgd,

L = length of pipeline between points i and j, feet,

and

a = b = constant.

For a two-member parallel network, the net benefit function becomes

$$NB_{p} = [Y_{1}*Q_{1} - m_{1}/2 * Q_{1}^{2} + Y_{2}*Q_{2} - m_{2}/2 * Q_{2}^{2}]$$
$$- [L_{S1}*a*Q_{1}^{b} + L_{S2}*a*Q_{2}^{b}]$$
(3-13)

To maximize net benefits, the partial derivatives with respect to  $Q_1$  and  $Q_2$  are taken and set equal to zero. Solving these equations determines the  $Q_1$ ,  $Q_2$  mix that maximizes net benefits. Therefore,

$$dNB_{p}/dQ_{1} = [Y_{1} - m_{1} * Q_{1}] - L_{S1}*a*b*Q_{1}^{(b-1)} = 0, \text{ or } (3-14)$$

$$P_{1} - MC_{S1} = 0, \text{ or } P_{1} = MC_{S1}$$

where  $MC_{S1}$  is the marginal cost associated with the pipeline between the source and user 1. Likewise,

$$dNB_{p}/dQ_{2} = [Y_{2} - m_{2} * Q_{2}] - L_{S2}*a*b*Q_{2}^{(b-1)} = 0, \text{ or}$$
(3-15)  
P\_{2} - MC\_{S2} = 0, or P\_{2} = MC\_{S2},

in general, 
$$P_i = MC_{Si}$$
 (3-16)

Series networks. (See Figure 3-2b) The analysis for this type of network is the same as for the parallel network. The only net benefit function change is with the cost portion of the equation. The two-member series network net benefit function becomes

$$NB_{s} = [Y_{1}*Q_{1} - m_{1}/2 * Q_{1}^{2} + Y_{2}*Q_{2} - m_{2}/2 * Q_{2}^{2}]$$
$$- [L_{S1}*a*(Q_{1}+Q_{2})^{b} + L_{12}*a*Q_{2}^{b}] \qquad (3-17)$$

Setting the partial derivatives with respect to  $Q_1$  and  $Q_2$  equal to zero and solving, gives

$$P_1 = MC_{S1}$$
, and  
 $P_2 = P_1 + MC_{12}$ 

which says, to maintain efficiency, user 1 is charged a per-unit charge equal to the marginal cost associated with the link between the source and him evaluated using the cumulative demand. While user 2's efficient price is equal to user 1's price plus the portion of the network marginal cost that is directly attributable to him.

In general, for efficient pricing in series networks

$$P_{k} = P_{(k-1)} + MC_{(k-1)k}$$
(3-18)

where

$$P_0 = 0.$$

Determining the maximum net benefit per-unit pricing for combinations of more than two users in a network only involves combining the appropriate series or parallel notions presented earlier (see Figure 3-2c). Consider the three user network [S1, 12, 13], source to user 1 and then to user 2 and also to user 3:

$$NB [S1,12,13] = [Y_1^*Q_1 - m_1/2 * Q_1^2 + Y_2^*Q_2 - m_2/2 * Q_2^2 + Y_3^*Q_3 - m_3/2 * Q_3^2] - [L_{S1}^*a^*(Q_1 + Q_2 + Q_3) * b + L_{12}^*a^*Q_2^*b + L_{13}^*a^*Q_3^*b]$$
(3-19)

then,

$$dNB/dQ_{1} = [Y_{1} - m_{1} * Q_{1}] - L_{S1}*a*b* (Q_{1} + Q_{2} + Q_{3})^{(b-1)} = 0, \text{ or } (3-20)$$

$$P_{1} - MC_{S1} = 0, \text{ or}$$

$$P_{1} = MC_{S1}, \text{ and} \qquad (3-21)$$

$$dNB_{dQ2} = [Y_2 - m_2 * Q_2] - L_{S1} * a * b * (Q_1 + Q_2 + Q_3)^{(b-1)} - L_{12} * a * b * Q_2^{(b-1)}$$
(3-22)

$$P_2 - MC_{S1} - MC_{12} = 0$$
, or  
 $P_2 = P_1 + MC_{12}$ , and (3-23)

$$dNB/dQ_{3} = [Y_{3} - m_{3} * Q_{3}] - L_{S1} * a^{*}b^{*} (Q_{1} + Q_{2} + Q_{3})^{(b-1)}$$
  
-  $L_{13} * a^{*}b^{*}Q_{3}^{(b-1)}$  (3-24)  
 $P_{3} - MC_{S1} - MC_{13} = 0, \text{ or}$ 

$$P_3 = P_1 + MC_{13}. \tag{3-25}$$

Observation of the optimally determined prices for this network of three users reveals  $P_2$  and  $P_3$  are simply extensions of the two-member series solution realizing, however, that  $MC_{S1}$  has changed since now all three users are sharing this link.

Just a final note before leaving this section. Should someone suggest uniform per-unit pricing for users in a given network, it can and will be demonstrated later by example that in fact the total net benefits of the network participants under uniform pricing are less in both the series and parallel cases. However, such an assumption does simplify determining the maximum net benefit flow mix.

#### Modeling Network

The following water supply network will be considered. Water supply source, S, is being designed to serve three users with demands of  $Q_1$ ,  $Q_2$ , &  $Q_3$ . Additionally, assume flow can go from user 1 to both user 2 and 3, and from user 2 to user 3. The water supply network showing all possible pipeline configurations and respective lengths of required pipe in feet is presented in Figure 3-3. The source can supply the total demand of the three users without facility expansion except for a new regional pipeline distribution network. The assumed user demand functions are as follows:

$$P_1 = 3 - 1.0^*Q_1$$
 (3-26)

P <sub>2</sub> = 5 - 1.25*Q <sub>2</sub>	(3-27)
--	--------

$$P_3 = 17.5 - 2.5^* Q_3 \tag{3-28}$$

where  $Q_i$  is the quantity demanded in million gallons per day, mgd, at a price,  $P_i$ , \$/1000gal. The marginal benefit or demand functions for each of the users are presented in Figure 3-4.

The total annualized cost function for constructing the pipelines are characterized by economies of scale, i.e., 0 < b < 1, and expressed as a linear function of distance and a nonlinear function of flow; or

$$C = a^* Q^{D^*} L_{ij} \tag{3-29}$$



Figure 3-3. Network model for water supply system example.



Figure 3-4. Demand functions for the users of the modeled water supply network.

where	C = total annualized pipeline cost, 1000\$/year,
	Q = quantity of flow, mgd,
	L = length of pipeline between points i and j, feet,
	a = constant=0.038
	b = constant=0.51.

Given these cost and demand functions, the objective is to determine the maximum net benefit network and associated costs for each user and each group of users in order to conduct the efficiency/equity analysis. The pricing criteria will be to use a variable per-unit charge for each user that maximizes the coalition's net benefit and then to apply a non-marginal fixed charge to ensure total project cost is covered.

### Net Benefits and Costs of the Various Coalitions

# "Go-alone" Net Benefits and Costs

To maximize net benefits for the individual members should they choose to pipe the water to themselves independently involves maximizing their respective benefits minus costs. The benefits,  $B_i$ , in 1000\$/year for each of the users, is simply the integral of their respective demand functions while their respective costs are the necessary pipeline costs to carry the user demand, that is (reference equation 3-3),

$$B_1 = (3^*Q_1 - 0.5^*Q_1^2) * 365$$
 (3-30)

$$B_2 = (5^*Q_2 - 0.625^*Q_2^2) * 365$$
 (3-31)

$$B_3 = (17.5^*Q_3 - 1.25^*Q_3^2) * 365$$
 (3-32)

where the constant, 365, converts daily to annual benefits. The associated annual costs calculated using equation 3-29 are shown below.

$$C_1 = 0.038^* Q_1^{0.51} * 17000 \tag{3-33}$$

$$C_2 = 0.038 * Q_2^{0.51} * 26000 \tag{3-34}$$

$$C_3 = 0.038 * Q_3^{0.51} * 30250 \tag{3-35}$$

As demonstrated earlier, pricing at marginal cost maximizes net benefits. For each user to go alone in this example, their marginal cost per-unit price and associated demands are

> $P_1 = 0.59$ \$/1000gal, for  $Q_1 = 2.41 \text{ mgd}$ ,  $P_2 = 0.76$ \$/1000gal, for  $Q_2 = 3.39 \text{ mgd}$ ,  $P_3 = 0.63$ \$/1000gal, for  $Q_3 = 6.75 \text{ mgd}$

The benefits versus costs curves for each of the users are shown in Figures 3-5a, 3-6 a, and 3-7a. The maximum net benefit points are identical to the intersection of the marginal benefits and marginal costs curves in Figures 3-5b, 3-6b, and 3-7b for users 1, 2, and 3 respectively. However, pricing at marginal cost results in total annual revenues not covering pipeline costs as tabulated in Table 3-1 and as depicted in Figures 3-8, 3-9, and 3-10. Total annual revenues, TR, are defined as

> TR = (P \* Q) \* c (3-36) TR = total annual revenues, 1000\$/year,

where P = price, \$/1000gal,

Q = mgd, and

c = conversion factor, 365 day/year

Therefore, if marginal cost pricing is to be maintained either deviation from marginal cost per-unit pricing must occur or some other means must be devised to recover these costs. The proposal is for a two-part tariff, using a marginal cost based variable per-unit charge and charging an additional



Figure 3-5. User 1 "go-alone" (a) benefits versus costs, and (b) demand versus marginal costs.



Figure 3-6. User 2 "go-alone" (a) benefits versus costs, and (b) demand versus marginal costs.



Figure 3-7. User 3 "go-alone" (a) benefits versus costs, and (b) demand versus marginal costs.













nonmarginal fixed fee to cover the remaining cost. For the go-alone option this results in a fixed charge fee of \$493, \$901, and \$1715 for the three users respectively. The variable and fixed charges and resulting maximum net benefit for each user are shown in Table 3-1.

It is worth pointing out that for such a nonmarginal charge to be acceptable it must result in a positive or break-even final net benefit for the user. This final net benefit for each member is used later for comparison to how well off each user is when joining a coalition. Each user will be required to be at least as well off as if he had gone alone. For the "go-alone" case the issue of equity does not apply since we assume each user's total revenues will have to cover total costs.

### Two-member Coalitions

For those coalitions with more than one member, the flow mix that maximizes net benefits for the coalition must be determined. The net benefit functions for the possible two member network configurations are:

Source to user 1, and then to user 2 [S1,12]

NB [S1,12] = 
$$(3^{*}Q_{1} - 0.5^{*}Q_{1}^{2} + 5^{*}Q_{2} - 0.625^{*}Q_{2}^{2}) * 365$$
  
-  $[a^{*}(Q_{1} + Q_{2})^{b^{*}}L_{S1} + a^{*}Q_{2}^{b^{*}}L_{12}]$  (3-37)

Source to user 1 and Source to user 2 [S1;S2]

NB [S1;S2] = 
$$(3^{*}Q_{1} - 0.5^{*}Q_{1}^{2} + 5^{*}Q_{2} - 0.625^{*}Q_{2}^{2})^{*} 365$$
  
-  $[a^{*}Q_{1}^{b*}L_{S1} + a^{*}Q_{2}^{b*}L_{S2}]$  (3-38)

Source to user 1, and then to user 3 [S1,13]

NB [S1,13] = 
$$(3^{*}Q_{1} - 0.5^{*}Q_{1}^{2} + 17.5^{*}Q_{3} - 1.25^{*}Q_{3}^{2}) * 365$$
  
-  $[a^{*}(Q_{1} + Q_{3})^{b*}L_{S1} + a^{*}Q_{3}^{b*}L_{13}]$  (3-39)

Table 3-1.	"Go-alone" user expected total revenues generated from
	marginal cost pricing with corresponding network costs
	and necessary annual charging system costs.

	Individual maximum	Qi	Pi	TRi	Ci	Annual	charging	system
User	net benefits	mgd	\$/1000gal	1000\$/yr	1000\$/yr	VC	FC	FC%
1	568	2.41	0.59	519	1012	519	493	49
2	1725	3.39	0.76	940	1841	940	901	49
3	19284	6.78	0.54	1336	3051	1336	1715	56

Note:

Qi = demand of user i which maximizes net benefits, mgd,

Pi = marginal cost price for user i, \$/1000gal,

TRi = user i expected annual revenue, 1000\$/yr,

Ci = cost of network for supplying Qi, 1000\$/yr,

VC = total annual variable charge at Pi,1000\$/yr, and

FC = total annual fixed charge equal to Ci - TRi, 1000\$/ yr.

Source to user 1 and Source to user 3 [S1;S3]

NB [S1;S3] = 
$$(3^{*}Q_{1} - 0.5^{*}Q_{1}^{2} + 17.5^{*}Q_{3} - 1.25^{*}Q_{3}^{2})^{*} 365$$
  
-  $[a^{*}Q_{1}^{b*}L_{S1} + a^{*}Q_{3}^{b*}L_{S3}]$  (3-40)

Source to user 2, and then to user 3 [S2,23]

NB [S2,23] = 
$$(5^{*}Q_{2} - 0.625^{*}Q_{2}^{2} + 17.5^{*}Q_{3} - 1.25^{*}Q_{3}^{2}) * 365$$
  
-  $[a^{*}(Q_{2} + Q_{3})^{b*}L_{S2} + a^{*}Q_{3}^{b*}L_{23}]$  (3-41)

Source to user 2 and Source to user 3 [S2;S3]

NB [S2;S3] = ( 
$$5^{*}Q_{2} - 0.625^{*}Q_{2}^{2} + 17.5^{*}Q_{3} - 1.25^{*}Q_{3}^{2}) * 365$$
  
- [ $a^{*}Q_{2}^{b^{*}}L_{S2} + a^{*}Q_{3}^{b^{*}}L_{S3}$ ] (3-42)

Each two-member net benefits equation is solved for the optimal  $Q_1$ ,  $Q_2$  or  $Q_1$ ,  $Q_3$  or  $Q_2$ ,  $Q_3$  flow mix by setting their respective partials equal to zero. The method of solution for the [S1,12] configuration is presented in Appendix D. All other two-member NB equations are solved in similar manner. The optimal network for each possible two-member coalition is considered the maximum net benefit configuration. A summary of the various optimal network configuration properties are presented in Table 3-2.

Once the flow mix that maximizes the coalition net benefit is determined, the variable portion of the two-part price to each user is based on their respective demand functions (see Figure 3-4). The appropriate curve is used at the determined optimal demand point and a per-unit price is determined. Setting each user's price equal to their marginal benefit, for coalitions with economies of scale, results in total project costs not being covered. It is easily verified that the resulting variable per-unit price to each user based on the optimal flow mix follows the price to marginal cost relationship developed earlier for series network configurations.

	per unit enargee let members et a given beament										
				ADJUSTED							
		Total	Individ	ual Net B	enefits	Individual demands			Price/gal		
Optimal		Coalition		1000\$/yr		MGD		\$/1000gal			
configuration	cost	max NB	User 1	User 2	User 3	Q1	Q2	Q3	Q1	Q2	Q3
[S1]	1013	567	567			2.41			0.59		
[S2]	1842	1724		1724			3.39			0.76	
[S3]	3051	19284			19284			6.78			0.54
[S1,12]	2567	2630	724	1905		2.46	3.57		0.54	0.54	
[S1,13]	4019	19935	567		19284	2.58		6.83	0.42		0.42
			•								
[S2,23]	4816	21125		1724	19284		3.56	6.78		0.55	0.55
[S1,12,13]	5337	22225	784	2157	19284	2.54	3.63	6.81	0.47	0.47	0.47

Table 3-3.	Table of optimal coalition configurations based on requiring uniform
	per-unit charges for members of a given coalition.

	ADJUSTED			ADJUSTED			
	F	ixed charg	je	Total annual cost			
Optimal		1000\$/yı	·		1000\$/yr		
configuration	User 1	User 2	User 3	User 1	User 2	User 3	
[S1]	493			1012	1		
[S2]		901			1842		
[S3]			1715			3051	
[S1,12]	380	999		865	1702		
[S1,13]	650		2018	1044		3060	
[S2,23]		1172	1699		1882	3051	
[S1,12,13]	389	845	1900	819	1461	3057	

As with the single member coalitions the remainder of the cost is to be recovered in a fixed annual charge. However, now a method must be chosen to apportion the required total annual charge in a manner acceptable to each of the participants. The remaining benefits method is used to apportion these costs with the ultimate stipulation that no user should lose net benefits with respect to his "go-alone" option. This stipulation requires a modification of the remaining benefits method to add this conditional check of what is known in game theory terms as individual rationality. In the case of two-member coalitions, should one member's net benefits decrease the decrease is made up by adjusting his annual charge downward by the amount necessary to make his net benefit at least remain constant. Accordingly, the annual charge of the other member of the coalition is raised this amount to ensure total revenues continue to cover total costs, or in game theory terms, group rationality is maintained. Mathematically, the allocation scheme is

Let

 $Q_i' = optimal flow for user i, mgd,$ 

 $P_i$  = marginal benefit price, \$/1000gal,

 $B_i$  = benefits of user i (see equations 3-30, 3-31,

and 3-32), 1000\$/yr,

then	$RB_{i} = B_{i} - (P_{i} * Q_{i}')$	(3-43)
where	RB <sub>i</sub> = remaining benefits for user i, 1000\$/yr,	
recall	TC = total cost of network for all users,	
	(see equation 3-12), 1000\$/yr, and	
	TR = total revenues produced, (see equation 3-36), 1000\$/yr,	
then	FC = TC - TR	(3-44)
where	FC = required fixed charge to cover costs, 1000\$/	′yr,

to apportion the required fixed charge,

	$FC_i = (RB_i / \sum RB_i) * FC$ , for all i	(3-45)
where	FC <sub>i</sub> = portion of fixed charge to user i, 1000\$/yr,	
finally,	$NB_i = B_i - P_i^*Q_j' - FC_i$	(3-46)
where	NB <sub>i</sub> = net benefits of user i, 1000\$/yr.	

As stated earlier, each user's net benefits must be greater than or equal to his "go-alone" net benefits. If all user's net benefits do not increase or remain the same, the following adjustment is made. The net benefits of the losing member is increased to at least his "go-alone" net benefits. This is done by decreasing his fixed charge by, his "go-alone" net benefits minus his current net benefits amount. Accordingly, a like amount is added to the other user's fixed charge. The resulting net benefits are then recalculated. Even though this may sound quite cumbersome, it is very easily set up using spreadsheet software and once a calculation template has been developed, recalculation for new conditions is quite simple.

#### Three-member Coalitions

The determination of the maximum net benefit flow mix for each of the possible three-member coalitions along with the determination of the variable per-unit charge is done essentially the same as for two-member coalitions with the exception that the net benefit functions change. The three-member coalition configurations to be considered and their respective net benefits functions are:

Source to 1, and then to user 2, and then to user 3 [S1,12,23]

NB [S1,12,23]=
$$(3^{*}Q_{1}-0.5^{*}Q_{1}^{2}+5^{*}Q_{2}-0.625^{*}Q_{2}^{2}$$
  
+17.5<sup>\*</sup>Q\_{3}-1.25<sup>\*</sup>Q\_{3}^{2})^{\*} 365 - [a^{\*}(Q\_{1}+Q\_{2}+Q\_{3})^{b\*}L\_{51}  
+ a<sup>\*</sup>(Q<sub>2</sub> + Q<sub>3</sub>)<sup>b\*</sup>L<sub>12</sub> + a<sup>\*</sup>Q\_{3}^{b\*}L\_{23}] (3-47)

Source to 1, and then to 2 and Source to 3 [S1,12;S3]

NB [S1,12;S3] = 
$$(3^{+}Q_{1}-0.5^{+}Q_{1}^{2}+5^{+}Q_{2}-0.625^{+}Q_{2}^{2}$$
  
+17.5^{+}Q\_{3}-1.25^{+}Q\_{3}^{2})^{\*} 365 - [a^{+}(Q\_{1}+Q\_{2})^{b\*}L\_{S1}  
+ a^{+}Q\_{2}^{b\*}L\_{12} + a^{+}Q\_{3}^{b\*}L\_{S3}] (3-48)

Source to 1, and then to 2, and also to 3 [S1,12,13]

NB [S1,12,13]=
$$(3^{*}Q_{1}-0.5^{*}Q_{1}^{2}+5^{*}Q_{2}-0.625^{*}Q_{2}^{2}$$
  
+17.5^{\*}Q\_{3}-1.25^{\*}Q\_{3}^{2})^{\*} 365 - [a^{\*}(Q\_{1}+Q\_{2}+Q\_{3})^{b^{\*}}L\_{S1}  
+ a^{\*}Q\_{2}^{b^{\*}}L\_{12} + a^{\*}Q\_{3}^{b^{\*}}L\_{13}] (3-49)

Source to 1, and then to 3, and Source to 2 [S1,13;S2]

NB [S1,13;S2] = 
$$(3^{+}Q_{1}-0.5^{+}Q_{1}^{2}+5^{+}Q_{2}-0.625^{+}Q_{2}^{2}$$
  
+17.5^{+}Q\_{3}-1.25^{+}Q\_{3}^{2})^{\*} 365 - [a^{+}(Q\_{1}+Q\_{3})^{b^{+}}L\_{S1}  
+  $a^{+}Q_{2}^{b^{+}}L_{S2}$  +  $a^{+}Q_{3}^{b^{+}}L_{13}$ ] (3-50)

Source to 1, Source to 2 and then to 3 [S1;S2,23]

NB [S1;S2,23] = 
$$(3^{\circ}Q_{1}-0.5^{\circ}Q_{1}^{2}+5^{\circ}Q_{2}-0.625^{\circ}Q_{2}^{2}$$
  
+17.5^{\circ}Q\_{3}-1.25^{\circ}Q\_{3}^{2})^{\*} 365 - [a^{\circ}Q\_{1}^{b^{\circ}L}S\_{1}  
+ a^{(Q\_{2}+Q\_{3})^{b^{\circ}L}S\_{2} + a^{\circ}Q\_{3}^{b^{\circ}L}S\_{2}] (3-51)

Source to 1, Source to 2, and Source to 3 [S1;S2;S3]

NB [S1;S2;S3] = 
$$(3^{\circ}Q_{1}-0.5^{\circ}Q_{1}^{2}+5^{\circ}Q_{2}-0.625^{\circ}Q_{2}^{2}$$
  
+17.5^{\circ}Q\_{3}-1.25^{\circ}Q\_{3}^{2})^{\*} 365 - [a^{\circ}Q\_{1}^{b^{\circ}L}S\_{1}  
+  $a^{\circ}Q_{2}^{b^{\circ}L}S_{2}$  +  $a^{\circ}Q_{3}^{b^{\circ}L}S_{3}$ ] (3-52)

Each of the three-member NB equations are solved for the optimal  $Q_1$ ,  $Q_2$ ,  $Q_3$ , flow mix by setting their respective partials equal to zero. The solution for the [S1,12,13] configuration is presented in Appendix D. All other three-member net benefit equations are solved in similar manner. Again, the optimal network for the three-member coalition is considered to be the maximum net benefit configuration. A summary of the properties of the optimal three-member network configuration [S1,12,13] is presented in Table 3-2. As with the two-member case, the reader can easily verify that the

resulting variable per-unit price to each user based on the optimal flow mix follows the price to marginal cost relationships developed earlier for networks involving both series and parallel components.

As with the two-member coalitions the remainder of the costs, after setting a variable per-unit price, is to be recovered in a fixed annual charge. Again the remaining benefits method is used to apportion these cost with the ultimate stipulation that no user should end up with his net benefit decreasing from his go-alone net benefit. In the case of three-member coalitions, should a member's net benefit decrease, it is made up by decreasing his annual charge by the amount necessary to make his net benefit at least remain constant. Respectively, the annual charge of the other members of the coalition is proportionally raised this amount to ensure total revenues continue to cover total costs. However, the allocation of the necessary addition to the other members requires some further fairness considerations. For fairness, a ratio of their individual net benefit gain to the total coalition net benefit gain is used as a proportionality factor for distributing the required fixed charge. Again this allocation process may sound cumbersome; however, it is very easy to set up using spreadsheet software and once a calculation template has been developed, recalculation for new conditions is quite simple.

# Further Discussion

After total enumeration of all the possible network pipeline configurations the game theoretic core principles must be checked. For each of the multiple member coalitions the characteristic cost and net benefit function is chosen as that configuration that maximizes net benefits. A summary of the optimal cost and maximum net benefit configurations based on using a <u>uniform</u> pricing scheme is presented in Table 3-3. The same summary of optimal costs and maximum net benefits for the various coalitions, while not requiring uniform per-unit charges but only that the coalition net benefit by maximized, are presented in Table 3-2. By comparison of the values in Tables 3-2 to those in 3-3, it is seen that using a uniform per-unit pricing reduces the various coalition net benefits that can be realized thru strictly efficient pricing.

Since the original intent was to maximize net benefits, the results of Table 3-2 are considered the optimal solution for the example network. Thus, the optimal cost functions, quantities demanded, and respective net benefit functions for each of the possible coalitions are

$$c(1) = 1013$$
 $c(2) = 1842$  $c(3) = 3050$  $c(12) = 2542$  $c(13) = 4002$  $c(23) = 4806$  $c(123) = 5306$ 

and,

$Q_1(12) = 2.63$ $Q_2(12) = 3.40$ $Q_3(13)$	= 6.72
$Q_1(13) = 2.70$ $Q_2(23) = 3.65$ $Q_3(23) = 3.65$	= 6.70
$Q_1(123) = 2.74$ $Q_2(123) = 3.49$ $Q_3(123)$	= 6.73

and,

NB(1) = 567	NB(2) = 1724	NB(3) = 19284
NB(12) = 2641	NB(13) = 19943	NB(23) = 21130
	NB(123) = 22240	

				ADJUSTE	D	-					
		Total	Individual Net Benefits		Individual demands		Price/gal				
Optimal		Coalition	1	1000\$/yr		MGD		\$/1000gal			
configuration	cost	max NB	User 1	User 2	User 3	Q1	Q2	Q3	Q1	Q2	Q3
[S1]	1013	567	567			2.41			0.59		
[S2]	1842	1724		1724			3.39			0.76	
[S3]	3050	19284			19284			6.78			0.54
[S1,12]	2542	2641	855	1786		2.63	3.40		0.37	0.76	
[S1,13]	4002	19943	567		19284	2.70		6.72	0.30		0.71
[S2,23]	4806	21130		1724	19284		3.65	6.70		0.44	0.76
[S1,12,13]	5306	22240	874	2082	19284	2.74	3.49	6.73	0.26	0.63	0.67

 Table 3-2.
 Table of optimal coalition configurations based on requiring efficient

 per-unit charges for members of a given coalition.

		ADJUSTE	D	ADJUSTED			
	F	ixed charg	je	Total annual cost			
Optimal		1000\$/yı	·	1000\$/yr			
configuration	User 1	User 1 User 2 User 3			User 2	User 3	
[S1]	493			1013			
[S2]		901			1841		
[S3]			1715			3051	
[S1,12]	404	843		762	1780		
[S1,13]	762		1289	1059		3035	
					•		
[S2,23]		1314	1167		1898	3030	
[S1,12,13]	499	705	1400	756	1510	3040	

For subadditivity and stability, the following core conditions must also be satisfied:

$$c(S) + c(T) \ge c(S \cup T),$$
  
NB(S) + NB(T)  $\le$  NB(S U T),

where S and T are any two disjoint sets of coalitions. The reader can verify that in fact both conditions hold for all coalitions and that the most efficient network is [S1,12,13].

However, this is not the end of the analysis. The individual member's net benefits must also have remained as well off or increased for each of the members to agree that what's good for all is also agreeable to them. Indeed, a check of Table 3-2 under each individual's net benefit column indicates that each has remained as well off or improved by joining the grand coalition versus if they went it alone. It would thus appear that this modified remaining benefits method for allocating cost can produce a charging algorithm that allows economically efficient pricing while also covering the total project costs through the use of an additional nonmarginal fee.

# Summary and Conclusions

The example presented of the three-member water supply network indicates that for the conditions as modeled, an equitable cost allocation can be devised through some simple modifications to the remaining benefits method. The proposed modifications to the remaining benefits method promotes the desired effect of participation in the optimal grand coalition network while allowing economically efficient pricing using a two-part scheme of a fixed nonmarginal charge and a variable per-unit charge. Indeed it has
been shown, that by taking advantage of the economies of scale groups can increase their overall net benefits and that, a sustainable two-part cost allocation scheme can be offered which allows fair and stable pricing.

One troubling observation concerning the solution for the network as modeled is that user 3's net benefit only remains constant. It may in fact be hard to convince him, under these circumstances, to participate solely for the good of the coalition. This result is believed due mainly to user 3's demand being so large relative to the demands of the other two players. Recommendations for future work are to consider this same scenerio with three users having demands more of the same magnitude and to explore alternative reconcilation schemes for apportioning the necessary fixed charge portion of the two-part tariff.

As stated in the beginning of this chapter, one should not overlook the many other problems, socio-political, etc, that will most definitely have to be dealt with when trying to set up regional based systems. On the other hand, the benefits to be gained through regional cooperation certainly warrant attention. Pricing at marginal cost is not a new concept but it is one that should continue to be explored for real world applications now that it is possible to demonstrate that stable and equitable pricing schemes can be devised.

#### CHAPTER 4 CONCLUSIONS

Many techniques are used to apportion the costs of a water resource project among the participants or purposes. Those techniques that are both efficient and equitable in their method of allocation are the desired techniques; however, it has only been in recent years that the issues of efficiency and equity have begun to be considered concurrently. The purpose of this thesis has been to show that these issues are interrelated and should therefore be considered concurrently. Two examples are presented in which the relationship between these issues is investigated, and it is demonstrated that stable cost allocations can be devised that allow the efficient system to be operated.

Chapter 2 presents an alternative form of cost allocation that an engineer could use to estimate the cost of removing a particular pollutant in a treatment plant which removes more than one type of pollutant. The dilemma addressed arises, for example, in an activated sludge unit that removes 5-day biochemical oxygen demand ( $BOD_5$ ), has luxury uptake of phosphorus, and removes suspended solids in the secondary clarifier. The question becomes "Should  $BOD_5$  be charged the whole cost of the activated sludge unit?" If not, then by what method should the cost be allocated among the various pollutants removed? This chapter presented a comparison of allocating costs based on a physical measure to the game theoretic Shapley value for a 20 MGD treatment plant designed to remove the four pollutants:  $BOD_5$ , total suspended solids (TSS), total phosphorus (Tot-P), and nitrogen (NH<sub>3</sub>-N). A

simplification of the Shapley value calculation is demonstrated which eases the tedious calculations of the Shapley value for large games. Using the simplified Shapley value, a fair set of pollutant charges is estimated without relying directly on a physical measure of use, such as pounds removed per day. This same cost allocation procedure would work for other combinations of pollutants to be removed in water and wastewater treatment plants including handling of hazardous waste. Even though a game theoretic approach to cost allocation involves hypothetical cost that must be estimated using cost models, it is still a viable cost allocation method that should not be overlooked.

In Chapter 3, an example of a three-member water supply network is used to show that for the conditions as modeled, an equitable and sustainable cost allocation can be devised through some simple modifications to the remaining benefits method. The proposed modifications to the remaining benefits method supports participation in the optimal grand coalition network while allowing economically efficient pricing using a two-part scheme of a fixed nonmarginal charge and a variable per-unit charge. It is shown, that by taking advantage of economies of scale, groups can increase their overall net benefits and that, a sustainable two-part cost allocation scheme can be offered which allows fair and stable pricing. Recommendations for future work are to consider this same scenerio with three users having demands more of the same magnitude and to explore alternative reconcilation schemes for apportioning the necessary fixed charge portion of the two-part tariff. Additionally, the network should be modeled unconstrained in the allowed flow direction between the network members. In the network as modeled, the flow was assumed only to occur from 1 to 2, or 1

to 3, or 2 to 3. This may be the very reason that user 3 was unable to benefit from the economies of scale that obviously provided users 1 and 2 a net benefit gain. The situation should be investigated where flow is also possible from 2 to 1, or 3 to 1, or 3 to 2.

One should not overlook the many other problems, socio-political, etc, that will have to be dealt with when trying to set up regional based systems. On the other hand, the benefits to be gained through regional networks certainly warrant attention. Pricing at marginal cost is not a new concept but it is one that should continue to be explored for real world applications now that it is possible to demonstrate that stable and equitable pricing schemes can be devised.

### APPENDIX A COST FUNCTION PARAMETER DOCUMENTATION

Appendix A contains the Chapter 2 supporting documentation for determining the cost function parameters based on fitting power functions to the data presented in Figures H-2 through H-13 of the EPA Areawide Assessment Procedures Manual Vol III. A spreadsheet software and personal computer were used to conduct all the necessary calculations.

Since the EPA cost data were from 1976, an Engineering News Record index factor is applied to the construction cost to adjust to 1986 dollars.

ENR index (Sept 76) = 2475 ENR index (Sept 87) = 4442.63

Adj. factor, 87 ENR/ 76 ENR = 1.80

For the amortization of the construction cost the following equation

is used:

Annual cost,  $C = Capital cost/[1-(1+i)^-n]/i$ 

where, Capital cost assumed to equal construction cost Amortization rate, i = 10%Years of amortization, n = 30

given these parameters,  $C = Construction \cos t / 9.43$ 

To determine the necessary power function parameters, points were picked off the figures at flows equal 0.1, 1.0, 10, and 100 MGD. The following transformation of the data was done;

using y = slope\*x + y\_intercept

then for  $y = a^*x^b$ , we see that

 $ln(y) = ln(a) + b^*ln(x)$ , or, by rearranging terms

 $\ln(a) = \ln(y) - b^* \ln(x)$ 

therefore, taking the exponential of both sides

 $a = exp[ln(y) - b^{*}ln(x)]$ , and b = the slope of a plotted line of ln(y) vs ln(x).

In the following pages, the "a" parameter for the construction cost portion of each cost curve is adjusted for current year 1986 and amortization of capital cost, i.e.;

a' = ENR factor \* a1/9.43

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Construction cost

Flow, MGD	<b>\$*10^6</b>					
X	у	ln (x)	ln (y)	b1	a1	a'
0.1	0.34	-2.30258509	-1.0788097	0.3716		0.1523
1	0.8	0	-0.2231436	0.3716	0.8	0.1523
1	0.8	0	-0.2231436	0.574		0.1523
10	3	2.302585093	1.09861229	0.574	0.8	0.1523
10	3	2.302585093	1.09861229	0.699		0.1142
100	15	4.605170186	2.7080502	0.699	0.6	0.1142

### O&M cost

Flow, MGD	\$*10^6				
X	У	ln (x) ·	In (y)	b2	a2
0.1	0.05	-2.30258509	-2.9957323	0.2553	0.09
1	0.09	0	-2.4079456	0.2553	0.09
	-				
1	0.09	0	-2.4079456	0.5229	0.09
10	0.3	2.302585093	-1.2039728	0.5229	0.09
10	0.3	2.302585093	-1.2039728	0.699	0.06
100	1.5	4.605170186	0.40546511	0.699	0.06

### Total cost

Flow, MGD	\$*10^6				
X	y*	ln (x)	In (y)	b	а
0.1	0.11474	-2.30258509	-2.1650845		
1	0.24233	0	-1.417455	0.325	0.242
1	0.24233	0	-1.417455		
10	0.87124	2.302585093	-0.1378408	0.556	0.242
10	0.87124	2.302585093	-0.1378408		
100	4.35619	4.605170186	1.47159713	0.699	0.174

\* Total cost, y = a' \*x^b1 + a2\*x^b2

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Construction cost

Flow, MGD	\$*10^6		<u>.</u>			
X	у	ln (x)	In (y)	b1	a1	a'
0.1	0.36	-2.30258509	-1.0216512	0.4075		0.1752
1	0.92	0	-0.0833816	0.4075	0.92	0.1752
1	0.92	0	-0.0833816	0.6383		0.1752
10	4	2.302585093	1.38629436	0.6383	0.92	0.1752
10	4	2.302585093	1.38629436	0.7202		0.1451
100	21	4.605170186	3.04452244	0.7202	0.7619	0.1451

### O&M cost

Flow, MGD	\$*10^6		,		
X	у	ln (x)	In (y)	b2	a2
0.1	0.06	-2.30258509	-2.8134107	0.368	0.14
1	0.14	0	-1.9661129	0.368	0.14
1	0.14	0	-1.9661129	0.5942	0.14
10	0.55	2.302585093	-0.597837	0.5942	0.14
10	0.55	2.302585093	-0.597837	0.8617	0.0756
100	4	4.605170186	1.38629436	0.8617	0.0756

### Total cost

Flow, MGD	\$*10^6				
X	у*	ln (x)	In (y)	b	а
0.1	0.12855	-2.30258509	-2.0514491		
1	0.31518	0	-1.1546131	0.389	0.315
1	0.31518	0	-1.1546131		
10	1.31165	2.302585093	0.27128578	0.619	0.315
10	1.31165	2.302585093	0.27128578		
100	7.99866	4.605170186	2.07927425	0.785	0.215

\* Total cost, y = a' \*x^b1 + a2\*x^b2

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Construction cost

Flow, MGD	\$*10^6					
X	У	ln (x)	In (y)	b1	a1	a'
0.1	0.44	-2.30258509	-0.8209806	0.4357		0.2285
1	1.2	0	0.18232156	0.4357	1.2	0.2285
1	1.2	0	0.18232156	0.574		0.2285
10	4.5	2.302585093	1.5040774	0.574	1.2	0.2285
10	4.5	2.302585093	1.5040774	0.7782		0.1428
100	27	4.605170186	3.29583687	0.7782	0.75	0.1428

### O&M cost

Flow, MGD	\$*10^6				
X	<b>y</b>	ln (x)	In (y)	b2	a2
0.1	0.06	-2.30258509	-2.8134107	0.3188	0.125
1	0.125	0	-2.0794415	0.3188	0.125
1	0.125	0	-2.0794415	0.5563	0.125
10	0.45	2.302585093	-0.7985077	0.5563	0.125
10	0.45	2.302585093	-0.7985077	0.7618	0.0779
100	2.6	4.605170186	0.95551145	0.7618	0.0779

## Total cost

Flow, MGD	\$*10^6				
X	у*	ln (x)	In (y)	b	а
0.1	0.14378	-2.30258509	-1.9394606		
1	0.35349	0	-1.0398861	0.391	0.353
1	0.35349	0	-1.0398861		
10	1.30686	2.302585093	0.26762432	0.568	0.353
10	1.30686	2.302585093	0.26762432		
100	7.74114	4.605170186	2.04654852	0.773	0.221

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\* Total cost, y = a' \*x^b1 + a2\*x^b2

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Construction cost

Flow, MGD	\$*10^6					
X	у	ln (x)	In (y)	b1	a1	a'
0.1	0.55	-2.30258509	-0.597837	0.4328		0.2837
1	1.49	0	0.39877612	0.4328	1.49	0.2837
1	1.49	0	0.39877612	0.605		0.2837
10	6	2.302585093	1.79175947	0.605	1.49	0.2837
10	6	2.302585093	1.79175947	0.8239		0.1714
100	40	4.605170186	3.68887945	0.8239	0.9	0.1714

### O&M cost

Flow, MGD	\$*10^6				
X	У	ln (x)	In (y)	b2	a2
0.1	0.06	-2.30258509	-2.8134107	0.426	0.16
1	0.16	0	-1.8325815	0.426	0.16
1	0.16	0	-1.8325815	0.688	0.16
10	0.78	2.302585093	-0.2484614	0.688	0.16
10	0.78	2.302585093	-0.2484614	0.8861	0.1014
100	6	4.605170186	1.79175947	0.8861	0.1014

### Total cost

Flow, MGD	\$*10^6				
X	y* -	ln (x)	In (y)	b	а
0.1	0.16473	-2.30258509	-1.8034666		
1	0.44371	0	-0.8125738	0.43	0.444
				· · ·	
1	0.44371	0	-0.8125738	,	
10	1.92247	2.302585093	0.65361331	0.637	0.444
10	1.92247	2.302585093	0.65361331		Ľ
100	13.6165	4.605170186	2.61128219	0.85	0.271

\* Total cost, y = a' \*x^b1 + a2\*x^b2

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Construction cost

Flow, MGD	\$*10^6					
X	у	ln (x)	ln (y)	b1	a1	a'
0.1	0.5	-2.30258509	-0.6931472	0.415		0.2475
1	1.3	0	0.26236426	0.415	1.3	0.2475
				-		
1	1.3	0	0.26236426	0.6185		0.2475
10	5.4	2.302585093	1.68639895	0.6185	1.3	0.2475
10	5.4	2.302585093	1.68639895	0.3504		0.4589
100	12.1	4.605170186	2.49320545	0.3504	2.4099	0.4589

### O&M cost

Flow, MGD	\$*10^6			•	
X	У	ln (x)	In (y)	b2	a2
0.1	0.06	-2.30258509	-2.8134107	0.426	0.16
1	0.16	0	-1.8325815	0.426	0.16
	<u>.</u>				
1	0.16	0	-1.8325815	0.5035	0.16
10	0.51	2.302585093	-0.6733446	0.5035	0.16
10	0.51	2.302585093	-0.6733446	0.8239	0.0765
100	3.4	4.605170186	1.22377543	0.8239	0.0765

#### Total cost

Flow, MGD	\$*10^6				
X	у*	ln (x)	ln (y)	b	а
0.1	0.15521	-2.30258509	-1.8630005		
1	0.40754	0	-0.8976255	0.419	0.408
1	0.40754	0	-0.8976255		
10	1.53823	2.302585093	0.43063066	0.577	0.408
10	1.53823	2.302585093	0.43063066		
100	5.70399	4.605170186	1.74116607	0.569	0.415

\* Total cost,  $y = a' *x^{b1} + a2^{*}x^{b2}$ 

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Construction cost

Flow, MGD	\$*10^6					
X	у	ln (x)	In (y)	b1	a1	a'
0.1	0.52	-2.30258509	-0.6539265	0.4601		0.2856
1	1.5	0	0.40546511	0.4601	1.5	0.2856
1	1.5	0	0.40546511	0.6092		0.2856
10	6.1	2.302585093	1.80828877	0.6092	1.5	0.2856
10	6.1	2.302585093	1.80828877	0.7332		0.2147
100	33	4.605170186	3.49650756	0.7332	1.1276	0.2147

### O&M cost

Flow, MGD	\$*10^6				
X	У	ln (x)	In (y)	b2	a2
0.1	0.06	-2.30258509	-2.8134107	0.5229	0.2
1.	0.2	0	-1.6094379	0.5229	0.2
1	0.2	0	-1.6094379	0.5441	0.2
10	0.7	2.302585093	-0.3566749	0.5441	0.2
10	0.7	2.302585093	-0.3566749	0.8539	0.098
100	5	4.605170186	1.60943791	0.8539	0.098

#### Total cost

Flow, MGD	\$*10^6				
X	у*	ln (x)	ln (y)	b	а
0.1	0.15901	-2.30258509	-1.83876		
1	0.48562	0	-0.7223315	0.485	0.486
1	0.48562	0	-0.7223315		
10	1.86152	2.302585093	0.62139123	0.584	0.486
10	1.86152	2.302585093	0.62139123		
100	11.2836	4.605170186	2.42335135	0.783	0.307

\* Total cost, y = a' \*x^b1 + a2\*x^b2

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Construction cost

Flow, MGD	\$*10^6					
X	у	ln (x)	In (y)	b1	a1	a'
0.1	0.52	-2.30258509	-0.6539265	0.4743		0.2951
1	1.55	0	0.43825493	0.4743	1.55	0.2951
1	1.55	0	0.43825493	0.595		0.2951
10	6.1	2.302585093	1.80828877	0.595	1.55	0.2951
10	6.1	2.302585093	1.80828877	0.6918		0.2362
100	30	4.605170186	3.40119738	0.6918	1.2403	0.2362

### O&M cost

Flow, MGD	\$*10^6				
X	У	ln (x)	In (y)	b2	a2
0.1	0.055	-2.30258509	-2.9004221	0.39	0.135
1	0.135	0	-2.0024805	0.39	0.135
1	0.135	0	-2.0024805	0.5857	0.135
10	0.52	2.302585093	-0.6539265	0.5857	0.135
10	0.52	2.302585093	-0.6539265	0.8155	0.0795
100	3.4	4.605170186	1.22377543	0.8155	0.0795

#### Total cost

Flow, MGD	\$*10^6	-			
X	y*	In (x)	In (y)	b	а
0.1	0.15401	-2.30258509	-1.8707086		
1	0.43014	0	-0.8436461	0.446	0.43
	-				
1	0.43014	0	-0.8436461		
10	1.68152	2.302585093	0.51969579	0.592	0.43
10	1.68152	2.302585093	0.51969579		
100	9.11237	4.605170186	2.20963326	0.734	0.31

\* Total cost, y = a' \*x^b1 + a2\*x^b2

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Construction cost

Flow, MGD	\$*10^6					
X	у	ln (x)	In (y)	b1	a1	a'
0.1	0.8	-2.30258509	-0.2231436	0.4191		0.3999
1	2.1	0	0.74193734	0.4191	2.1	0.3999
1	2.1	0	0.74193734	0.6223		0.3999
10	8.8	2.302585093	2.17475172	0.6223	2.1	0.3999
10	8.8	2.302585093	2.17475172	0.7798		0.2782
100	53	4.605170186	3.97029191	0.7798	1.4611	0.2782

### O&M cost

Flow, MGD	\$*10^6				
X	у	ln (x)	In (y)	b2	a2
0.1	0.09	-2.30258509	-2.4079456	0.368	0.21
1	0.21	0	-1.5606477	0.368	0.21
1	0.21	0	-1.5606477	0.5809	0.21
10	0.8	2.302585093	-0.2231436	0.5809	0.21
10	0.8	2.302585093	-0.2231436	0:8293	0.1185
100	5.4	4.605170186	1.68639895	0.8293	0.1185

### Total cost

Flow, MGD	\$*10^6				
X	у* .	ln (x)	ln (y)	b	a
0.1	0.24233	-2.30258509	-1.417455		
1	0.60987	0	-0.4945157	0.401	0.61
1	0.60987	0	-0.4945157		
10	2.47563	2.302585093	0.90649478	0.608	0.61
10	2.47563	2.302585093	0.90649478		
100	15.4919	4.605170186	2.74031477	0.796	0.396

\* Total cost, y = a' \*x^b1 + a2\*x^b2

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Construction cost

Flow, MGD	\$*10^6					
X	у	ln (x)	In (y)	b1	a1	a'
0.1	0.48	-2.30258509	-0.7339692	0.5492		0.3237
1	1.7	0	0.53062825	0.5492	1.7	0.3237
1	1.7	0	0.53062825	0.6726		0.3237
10	8	2.302585093	2.07944154	0.6726	1.7	0.3237
10	8	2.302585093	2.07944154	0.699		0.3047
100	40	4.605170186	3.68887945	0.699	1.6	0.3047

### O&M cost

Flow, MGD	\$*10^6				
X	У	ln (x)	In (y)	b2	a2
0.1	0.07	-2.30258509	-2.65926	0.4102	0.18
1	0.18	0	-1.7147984	0.4102	0.18
1	0.18	0	-1.7147984	0.6478	0.18
10	0.8	2.302585093	-0.2231436	0.6478	0.18
10	0.8	2.302585093	-0.2231436	0.8373	0.1164
100	5.5	4.605170186	1.70474809	0.8373	0.1164

### Total cost

Flow, MGD	\$*10^6				
X	<b>y*</b> -	ln (x)	ln (y)	b	а
0.1	0.1614	-2.30258509	-1.823882		
1	0.5037	0	-0.6857721	0.494	0.504
1	0.5037	0	-0.6857721		
10	2.3233	2.302585093	0.84298847	0.664	0.504
10	2.3233	2.302585093	0.84298847		·
100	13.1165	4.605170186	2.57387087	0.752	0.412

\* Total cost, y = a' \*x^b1 + a2\*x^b2

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Construction cost

Flow, MGD	\$*10^6					
X	у	In (x)	ln (y)	b1	a1	a'
0.1	0.5	-2.30258509	-0.6931472	0.5315		0.3237
1	1.7	0	0.53062825	0.5315	1.7	0.3237
				-		
1	1.7	0	0.53062825	0.6726		0.3237
10	8	2.302585093	2.07944154	0.6726	1.7	0.3237
10	8	2.302585093	2.07944154	0.699		0.3047
100	40	4.605170186	3.68887945	0.699	1.6	0.3047

### O&M cost

Flow, MGD	\$*10^6				
X	У	ln (x)	In (y)	b2	a2
0.1	0.17	-2.30258509	-1.7719568	0.5757	0.64
1	0.64	0	-0.4462871	0.5757	0.64
1	0.64	0	-0.4462871	0.6709	0.64
10	3	2.302585093	1.09861229	0.6709	0.64
10	3	2.302585093	1.09861229	0.8239	0.45
100	20	4.605170186	2.99573227	0.8239	0.45

### Total cost

Flow, MGD	\$*10^6				
X	у*	ln (x)	In (y)	b	а
0.1	0.26521	-2.30258509	-1.3272475		
1	0.9637	0	-0.036974	0.56	0.964
1	0.9637	0	-0.036974		
10	4.5233	2.302585093	1.50924175	0.672	0.964
10	4.5233	2.302585093	1.50924175		
100	27.6165	4.605170186	3.31841337	0.786	0.741

\* Total cost,  $y = a' *x^b1 + a2*x^b2$  ·

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Construction cost

Flow, MGD	\$*10^6					
X	у	ln (x)	In (y)	b1	a1	a'
0.1	0.82	-2.30258509	-0.1984509	0.5775		0.5903
1	3.1	0	1.13140211	0.5775	3.1	0.5903
1	3.1	0	1.13140211	0.55		0.5903
10	11	2.302585093	2.39789527	0.55	3.1	0.5903
10	11	2.302585093	2.39789527	0.7648		0.36
100	64	4.605170186	4.15888308	0.7648	1.8906	0.36

### O&M cost

Flow, MGD	\$*10^6				
X	У	ln (x)	In (y)	b2	a2
0.1	0.105	-2.30258509	-2.2537949	0.3768	0.25
1	0.25	0 .	-1.3862944	0.3768	0.25
1	0.25	0	-1.3862944	0.6435	0.25
10	1.1	2.302585093	0.09531018	0.6435	0.25
10	1.1	2.302585093	0.09531018	0.9128	0.1344
100	9	4.605170186	2.19722458	0.9128	0.1344

### Total cost

Flow, MGD	\$*10^6				
X	y*	ln (x)	In (y)	b	а
0.1	0.26114	-2.30258509	-1.3427054		
1	0.84028	0	-0.1740217	0.508	0.84
1	0.84028	0	-0.1740217		
10	3.19454	2.302585093	1.1614422	0.58	0.84
10	3.19454	2.302585093	1.1614422		
100	21.1864	4.605170186	3.05335936	0.822	0.482

\* Total cost, y = a' \*x^b1 + a2\*x^b2

\_\_\_\_

Construction cost

Flow, MGD	\$*10^6					
X	у	ln (x)	In (y)	b1	a1	a'
0.1	0.9	-2.30258509	-0.1053605	0.5229		0.5712
1	3	0	1.09861229	0.5229	3	0.5712
1	3	0	1.09861229	0.699		0.5712
10	15	2.302585093	2.7080502	0.699	3	0.5712
10	15	2.302585093	2.7080502	0.7377		0.5225
100	82	4.605170186	4.40671925	0.7377	2.7439	0.5225

### O&M cost

Flow, MGD	\$*10^6				
x	У	ln (x)	ln (y)	b2	a2
0.1	0.13	-2.30258509	-2.0402208	0.4658	0.38
. 1	0.38	0	-0.967584	0.4658	0.38
1	0.38	0	-0.967584	0.6243	0.38
10	1.6	2.302585093	0.47000363	0.6243	0.38
10	1.6	2.302585093	0.47000363	0.9098	0.1969
100	13	4.605170186	2.56494936	0.9098	0.1969

### Total cost

Flow, MGD	\$*10^6				
X	у*	ln (x)	In (y)	b	a
0.1	0.30137	-2.30258509	-1.1994125		
1	0.95124	0	-0.0499916	0.499	0.951
1	0.95124	0	-0.0499916		
10	4.45619	2.302585093	1.49429346	0.671	0.951
10	4.45619	2.302585093	1.49429346		
100	28.6138	4.605170186	3.35388989	0.808	0.694

\* Total cost, y = a' \*x^b1 + a2\*x^b2

#### APPENDIX B TREATMENT SYSTEM REMOVAL EFFICIENCIES AND COST PARAMETERS

Appendix B presents the subsets of the twelve treatment systems discussed in Chapter 2 able to meet the required removal efficiencies for all possible coalitions. For each subset, the coalition cost function parameters and total system treatment cost are given. The treatment conditions are:

		indent concentrati	
Pollutant	# Description	(mg/L)	
1	BOD5_mgL_IN	210	
2	SS_mgL_IN	230	<b>Q</b> = 20 MGD
3	TOT_P_mgL_IN	11	
4	N_mgL_IN	20	• ·
	•		

### Influent concentration

# Effluent concentration

	(mg/L)
BOD5_mgL_EFF	10
SS_mgL_EFF	5
TOT_P_mgL_EFF	0.5
N_mgL_EFF	1.9

	Required	%removal
BOD5_REM	0.	95
SS_REM	0.	98
TOT_P_REM	0.	95
N_REM	0.	91

The possible coalitions, C(i), where i=1,2,3, or 4 and 1=BOD5, 2=TSS, 3=TOT\_P, 4=NH3\_N are:

	I	C(12)	BOD5	TSS				
		1	>=0.95	>=0.98				
C(1)	BOD5	1		Ĩ	C(123)	BOD5	TSS	TOT_P
	>=0.95	C(13)	BOD5	TOT_P		>=0.95	>=0.98	>=0.95
	0.95	1	>=0.95	>=0.95				
C(2)	TSS	ł		1	C(124)	BOD5	TSS	NH3_N
	>=0.98	C(14)	BOD5	NH3_N	1	>=0.95	>=0.98	>=0.91
	0.98	ļ	>=0.95	>=0.91			-	
C(3)	TOT_P				C(134)	BOD5	TOT_P	NH3_N
	>=0.95	C(23)	TSS	TOT_P		>=0.95	>=0.95	>=0.91
	0.95	I	>=0.98	>=0.95				
C(4)	NH3_N				C(234)	TSS	TOT_P	NH3_N
	>=0.91	C(24)	TSS	NH3_N		>=0.98	>=0.95	>=0.91
	0.91	l	>=0.98	>=0.91				
	l							
	l	C(34)	TOT_P	NH3_N	C(1234)	BOD5	TSS	TOT_P NH3_N
			>=0.95	>=0.91		>=0.95	>=0.98	>=0.95 >=0.91

(see Note 1)		Pollu	utant			Annual cost parameters			
System	R	emoval Ef	ficiencies				(see Note 2)		
No.	BOD	5 TSS	TOT_P	NH3_N	•	а	b	aQ^b	
.1	0.38	0.57	0.18	0.00		0.174	0.699	\$1,410,000	
2	0.52	0.78	0.82	0.00		0.215	0.785	\$2,260,000	
3	0.79	0.74	0.27	0.10		0.221	0.773	\$2,230,000	
4	0.88	0.87	0.82	0.10		0.271	0.85	\$3,470,000	
5	0.90	0.91	0.36	0.15		0.415	0.569	\$2,280,000	
6	0.93	0.93	0.82	0.15		0.307	0.783	\$3,200,000	
7	0.95	0.91	0.27	0.90		0.31	0.734	\$2,800,000	
8	0.95	0.91	0.27	0.95		0.396	0.796	\$4,300,000	
9	0.95	0.96	0.91	0.90		0.412	0.752	\$3,910,000	
10	0.98	0.98	0.91	0.00		0.741	0.786	\$7,800,000	
11	0.98	0.98	0.95	0.95		0.482	0.822	\$5,650,000	
12	0.99	0.98	0.95	0.95		0.694	0.808	\$7,800,000	

The twelve treatment system removal efficiencies and cost parameters based on 20 MGD are:

Note 1: Treatment systems include disinfection, sludge handling, miscellaneous structures, and support personnel. Note 2: Use Q in MGD and then multiply aQ<sup>b</sup> by 1,000,000.

The possible subset of treatment systems for each coalition are:

		Pollu	tant				
System	F	Removal	Efficienc	ies	Ann	ual cost	parameters
No.	BOD5	TSS	TOT_P	NH3_N	а	b	aQ^b
7	0.95	0.91	0.27	0.90	0.31	0.734	\$2,800,000
8	0.95	0.91	0.27	0.95	0.396	0.796	\$4,300,000
9	0.95	0.96	0.91	0.90	0.412	0.752	\$3,910,000
10	0.98	0.98	0.91	0.00	0.741	0.786	\$7,800,000
11	0.98	0.98	0.95	0.95	0.482	0.822	\$5,650,000
12	0.99	0.98	0.95	0.95	0.694	0.808	\$7,800,000

1. Treatment system data on those systems meeting coalition c(1) criteria.

2. Treatment system data on those systems meeting coalition c(2) criteria.

		Pollu	tant				
System	F	Removal	Efficienc	ies	Ann	ual cost	parameters
No.	BOD5	TSS	TOT_P	NH3_N	а	b	aQ^b
10	0.98	0.98	0.91	0.00	0.741	0.786	\$7,800,000
11	0.98	0.98	0.95	0.95	0.482	0.822	\$5,650,000
12	0.99	0.98	0.95	0.95	0.694	0.808	\$7,800,000

3. Treatment system data on those systems meeting coalition c(3) criteria.

		Pollu	tant			· · ·		
System	Removal Efficiencies					Ann	ual cost	parameters
No.	BOD5	TSS	TOT_P	NH3_N		а	b	aQ^b
11	0.98	0.98	0.95	0.95		0.482	0.822	\$5,650,000
12	0.99	0.98	0.95	0.95		0.694	0.808	\$7,800,000

4. Treatment system data on those systems meeting coalition c(4) criteria.

		Pollu	tant				
System	F	Removal	Efficienc	ies	Ann	ual cost	parameters
No.	BOD5	TSS	TOT_P	NH3_N	а	b	aQ^b
8	0.95	0.91	0.27	0.95	0.396	0.796	\$4,300,000
11	0.98	0.98	0.95	0.95	0.482	0.822	\$5,650,000
12	0.99	0.98	0.95	0.95	0.694	0.808	\$7,800,000

		Pollu	tant				
System	F	Removal	Efficienc	ies	Ann	ual cost	parameters
No.	BOD5	TSS	TOT_P	NH3_N	а	b	aQ^b
10	0.98	0.98	0.91	0.00	0.741	0.786	\$7,800,000
11	0.98	0.98	0.95	0.95	0.482	0.822	\$5,650,000
12	0.99	0.98	0.95	0.95	0.694	0.808	\$7,800,000

5. Treatment system data on those systems meeting coalition c(12) criteria.

6. Treatment system data on those systems meeting coalition c(13) criteria.

		Pollu	tant					
System	Removal Efficiencies					Ann	ual cost	parameters
No.	BOD5	TSS	TOT_P	NH3_N		а	b	aQ^b
11	0.98	0.98	0.95	0.95		0.482	0.822	\$5,650,000
12	0.99	0.98	0.95	0.95		0.694	0.808	\$7,800,000

7. Treatment system data on those systems meeting coalition c(14) criteria.

		Pollu	tant				
System	F	Removal	Efficienc	ies	Ann	ual cost	parameters
No.	BOD5	TSS	TOT_P	NH3_N	а	b	aQ^b
8	0.95	0.91	0.27	0.95	0.396	0.796	\$4,300,000
11	0.98	0.98	0.95	0.95	0.482	0.822	\$5,650,000
12	0.99	0.98	0.95	0.95	0.694	0.808	\$7,800,000

8. Treatment system data on those systems meeting coalition c(23) criteria.

		Pollu	tant				
System	F	Removal	Efficienc	ies	Ann	ual cost	parameters
No.	BOD5	TSS	TOT_P	NH3_N	а	b	aQ^b
11	0.98	0.98	0.95	0.95	0.482	0.822	\$5,650,000
12	0.99	0.98	0.95	0.95	0.694	0.808	\$7,800,000

		Pollu	tant				
System	F	Removal	Efficienc	cies	Ann	ual cost	parameters
No.	BOD5	TSS	TOT_P	NH3_N	а	b	aQ^b
11	0.98	0.98	0.95	0.95	0.482	0.822	\$5,650,000
12	0.99	0.98	0.95	0.95	0.694	0.808	\$7,800,000

9. Treatment system data on those systems meeting coalition c(24) criteria.

10. Treatment system data on those systems meeting coalition c(34) criteria.

		Pollu	tant				
System	F	Removal	Efficienc	ies	Ann	ual cost	parameters
No.	BOD5	TSS	TOT_P	NH3_N	а	b	aQ^b
11	0.98	0.98	0.95	0.95	0.482	0.822	\$5,650,000
12	0.99	0.98	0.95	0.95	0.694	0.808	\$7,800,000

11. Treatment system data on those systems meeting coalition c(123) criteria.

		Pollu	tant				
System	F	Removal	Efficienc	ies	Ann	ual cost	parameters
No.	BOD5	TSS	TOT_P	NH3_N	а	b	aQ^b
11	0.98	0.98	0.95	0.95	0.482	0.822	\$5,650,000
12	0.99	0.98	0.95	0.95	0.694	0.808	\$7,800,000

12. Treatment system data on those systems meeting coalition c(124) criteria.

		Pollu	tant				
System	F	Removal	Efficienc	ies	Ann	ual cost	parameters
No.	BOD5	TSS	TOT_P	NH3_N	а	b	aQ^b
11	0.98	0.98	0.95	0.95	0.482	0.822	\$5,650,000
12	0.99	0.98	0.95	0.95	0.694	0.808	\$7,800,000

		Pollu	tant				
System	F	Removal	Efficienc	ies	Ann	ual cost	parameters
No.	BOD5	TSS	TOT_P	NH3_N	а	b	aQ^b
11	0.98	0.98	0.95	0.95	0.482	0.822	\$5,650,000
12	0.99	0.98	0.95	0.95	0.694	0.808	\$7,800,000

13. Treatment system data on those systems meeting coalition c(134) criteria.

14. Treatment system data on those systems meeting coalition c(234) criteria.

		Pollu	tant				
System	F	Removal	Efficienc	ies	Ann	ual cost	parameters
No.	BOD5	TSS	TOT_P	NH3_N	а	b	aQ^b
11	0.98	0.98	0.95	0.95	0.482	0.822	\$5,650,000
12	0.99	0.98	0.95	0.95	0.694	0.808	\$7,800,000

15. Treatment system data on those systems meeting coalition c(1234) criteria.

		Pollu	tant				
System	F	Removal	Efficienc	ies	Ann	ual cost	parameters
No.	BOD5	TSS	TOT_P	NH3_N	а	b	aQ^b
11	0.98	0.98	0.95	0.95	0.482	0.822	\$5,650,000
12	0.99	0.98	0.95	0.95	0.694	0.808	\$7,800,000

APPENDIX C MARGINAL COSTS OF POLLUTANT REMOVAL

Appendix C contains the marginal cost data for each of the pollutants considered in Chapter 2 based on the pollutants relative position when joining the coalition.

loine			BOD5	- Marg	inal co	ost (\$*	10^6)		
301115	BOD5 TSS Tot-P NH3-N								
	В	OD5	TSS		Tot-P			NH3-N	
1st	na*	na	2.80	2.80	2.80	2.80	2.80	2.80	
					After				
	В	OD5	٦	rss		Tot-P		NH3-N	
2nd	na	na	0.00	0.00	0.00	0.00	0.00	0.00	
3rd	na	na	0.00	0.00	0.00	0.00	0.00	0.00	
4th	na	na	0.00	0.00	0.00	0.00	0.00	0.00	

\*na = none applicable

Joins			TSS -	Margin	al cos efore-	t (\$*1	0^6)			
	В	OD5	7	rss	Tot-P		NH3-N			
1st	5.65	5.65	na na		5.65	5.65	5.65	5.65		
		5.65 5.65 na na 5.65 5.65 5.65 5.65								
							NH3-N			
	В	OD5	1	rss		Tot-P		NH3-N		
2nd	<b>B</b> 2.85	<b>OD5</b> 2.85	na 1	r <b>SS</b> na	0.00	Tot-P 0.00	1.35	NH3-N 1.35		
2nd	<b>B</b> 2.85	<b>OD5</b> 2.85	na na	r <b>SS</b> na	0.00	Tot-P 0.00	1.35	NH3-N 1.35		
2nd 3rd	8 2.85 1.35	OD5 2.85 0.00	na na	r <b>SS</b> na na	0.00	<b>Tot-P</b> 0.00 0.00	1.35 1.35	NH3-N 1.35 0.00		
2nd 3rd	8 2.85 1.35	OD5 2.85 0.00	na na	na na	0.00	Tot-P 0.00 0.00	1.35	NH3-N 1.35 0.00		

loine			Tot-P	Margi	nal co	st (\$* <sup>-</sup>	10^6)	
00113					eloie-			
	B	OD5	1	SS	-	Γot-P		NH3-N
1st	5.65	5.65	5.65	5.65	na	na	5.65	5.65
					After			
	В	OD5	1	ISS		Tot-P		NH3-N
2nd	<b>B</b> 2.85	<b>OD5</b> 2.85	۲ 0.00	<b>SS</b> 0.00	na	rot-P na	1.35	<b>NH3-N</b> 1.35
2nd	<b>B</b> 2.85	<b>OD5</b> 2.85	0.00	<b>SS</b> 0.00	na	rot-P na	1.35	NH3-N 1.35
2nd 3rd	<b>B</b> 2.85 1.35	OD5 2.85 0.00	0.00 0.00	<b>SS</b> 0.00 0.00	na na	na	1.35	NH3-N 1.35 0.00
2nd 3rd	<b>B</b> 2.85 1.35	OD5 2.85 0.00	0.00 0.00	0.00 0.00	na na	na na	1.35 1.35	NH3-N 1.35 0.00

Joins			NH3-N	- Marg	ginal c efore-	ost (\$	*10^6)		
001110	B	OD5	1	rss		Tot-P	NH3-N		
1st	4.30	4.30	4.30	4.30	4.30	4.30	na	na	
					A 64 a x				
					Aller				
	В	OD5	1	rss		Гot-Р		NH3-N	
2nd	<b>B</b> 1.50	<b>OD5</b> 1.50	۲ 0.00	0.00	0.00	<b>Fot-P</b> 0.00	na	NH3-N na	
2nd	<b>B</b> 1.50	OD5 1.50	0.00	0.00	0.00	<b>Fot-P</b> 0.00	na	NH3-N na	
2nd 3rd	<b>B</b> 1.50 0.00	OD5 1.50 0.00	0.00	0.00 0.00	0.00	<b>Fot-P</b> 0.00 0.00	na na	NH3-N na na	
2nd 3rd	<b>B</b> 1.50 0.00	OD5 1.50 0.00	0.00 0.00	0.00 0.00	0.00	<b>Fot-P</b> 0.00 0.00	na na	NH3-N na na	

### APPENDIX D SOLUTIONS TO NET BENEFITS EQUATIONS

This appendix contains the solution to equations 3-37 and 3-49 for determining the flow mix that maximizes net benefits for the particular coalition arrangement.

Equation 3-37: Net benefits for [S1,12]  
1. NB[S1,12] = 
$$[3^{*}Q_{1}^{-.5^{*}}Q_{1}^{-2+5^{*}}Q_{2}^{-.625^{*}}Q_{2}^{-2}]$$
  
 $- [L_{S1}^{*}a^{*}(Q_{1}+Q_{2})^{b} + L_{12}^{*}a^{*}Q_{2}^{-b}]$ 

To solve for the optimal flow mix that maximizes the net benefits equation the partial derivatives of the NB equation with respect to  $Q_1$  and  $Q_2$  are each taken and set equal to zero:

2. 
$$dNB/dQ_1 = 1095 - 365^*Q_1 - L_{S1}^*a^*b^*(Q_1 + Q_2)^{(b-1)} = 0$$

3. 
$$dNB/dQ_2 = 1825 - 456.25^*Q_2 - L_{S1}^*a^*b^*(Q_1 + Q_2)^{(b-1)}$$
  
-  $L_{12}^*a^*b^*Q_2^{(b-1)} = 0$ 

Subtracting equation 3 from equation 2 gives

4. 730 + 365<sup>\*</sup>Q<sub>1</sub> - 456.25<sup>\*</sup>Q<sub>2</sub> - 
$$L_{12}$$
<sup>\*</sup>a<sup>\*</sup>b<sup>\*</sup>Q<sub>2</sub><sup>^</sup>(b-1) = 0

Using equation 4,  $Q_1$  can be solved for in terms of  $Q_2$ , or,

5. 
$$Q_1 = [(L_{12}^*a^*b^*Q_2^{(b-1)})/365] + 1.25^*Q_2 - 2$$

Equation 5 for  $Q_1$  is substituted into equation 1 and through the use of spreadsheet software, the optimal  $Q_2$  is solved for using an iterative technique. Once  $Q_2$  is known, the corresponding  $Q_1$  can be determined using equation 5.

Equation 3-49: Net benefits for [S1,12,13]  
1. NB[S1,12,13] = 
$$[3^{Q}_{1}-.5^{Q}_{1}^{2}+5^{Q}_{2}-.625^{Q}_{2}^{2}+17.5^{Q}_{3}-1.25^{Q}_{3}^{2}]$$
  
 $- [L_{S1}^{*}a^{*}(Q_{1}+Q_{2}+Q_{3})^{b} + L_{12}^{*}a^{*}Q_{2}^{b} + L_{13}^{*}a^{*}Q_{3}^{b}]$ 

To solve for the optimal flow mix that maximizes the three member net benefits equation, the partial derivatives of the NB equation with respect to  $Q_1$ ,  $Q_2$ , and  $Q_3$  are each taken and set equal to zero:

2. 
$$dNB/dQ_1 = 1095 - 365^*Q_1 - L_{S1}^*a^*b^*(Q_1 + Q_2 + Q_3)^{(b-1)} = 0$$

3. 
$$dNB/dQ_2 = 1825 - 456.25^*Q_2 - L_{S1}^*a^*b^*(Q_1 + Q_2 + Q_3)^{(b-1)}$$
  
-  $L_{12}^*a^*b^*Q_2^{(b-1)} = 0$ 

4. 
$$dNB/dQ_3 = 6387.5 - 912.5^*Q_3 - L_{S1}^*a^*b^*(Q_1 + Q_2 + Q_3)^{(b-1)}$$
  
-  $L_{13}^*a^*b^*Q_3^{(b-1)} = 0$ 

Subtracting equation 4 from equation 2 gives

5. 
$$5292.5 + 365^{\circ}Q_{1} - 912.5^{\circ}Q_{3} - L_{13}^{\circ}a^{\circ}b^{\circ}Q_{3}^{\circ}(b-1) = 0$$

Using equation 5,  $Q_1$  can be solved for in terms of  $Q_3$ , or,

6. 
$$Q_1 = [(L_{13}^*a^*b^*Q_3^{(b-1)})/365] + 2.5^*Q_3 - 14.5]$$

Using equation 2,  $Q_2$  can be solved for in terms of  $Q_1$ , and  $Q_3$ , or,

7. 
$$Q_2 = [(1095 - 365 Q_1)/(L_{S1} a b)]^{(1/(b-1))} - Q_1 - Q_3$$

Equations 6 and 7 for  $Q_1$  and  $Q_2$  respectively are substituted into equation 1 and through the use of spreadsheet software, the optimal  $Q_3$  is solved for using an iterative technique. Then by back substitution into equations 6 and 7,  $Q_1$ and  $Q_2$  are solved for respectively giving the final flow mix that maximizes net benefits.

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## **BIOGRAPHICAL SKETCH**

Stephen Norman Payne was born March 11, 1954, in Toccoa, Georgia, the son of Annie Pearl Payne and Clarence Norman Payne. He graduated from Stephens County High school located in Toccoa, Georgia, in 1972 as class valedictorian. He attended the Georgia Institute of Technology in Atlanta, Georgia, from 1972 to 1976, graduating magna cum laude while receiving a Bachelor of Science in industrial and systems engineering. Subsequently, he worked for the Tennessee Valley Authority as a safety engineer and, later for Coats and Clark, Industries as an industrial engineer. He was commissioned in the U.S. Air Force (USAF) in 1979. Since entering the USAF, he has held assignments at the USAF Occupational and Environmental Health Laboratory, Brooks AFB, Texas (Consultant, Bioenvironmental Engineer) and at the Base Bioenvironmental Engineering office, Robins AFB, Georgia (Base Bioenvironmental Engineer). He was selected by the Air Force Institute of Technology (AFIT) in 1985 for the master of engineering program in environmental engineering. In 1986, he entered the University of Florida (UF) to pursue the Master of Engineering degree. He graduated from UF in 1988 with a Master of Engineering in environmental engineering. Upon graduation, he and his wife, Debbie, moved to Hickam AFB, Hawaii.

100